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DEVELOPMENTS IN USSR CYBERNETICS

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CYBERNETIC MODELING OF THOUGHT

[Article by Professor A. Spirkin, Doctor of Philosophical Sciences, Moscow; Minsk, Kommunist Belorussii, Russian, No 11, November 1968, pp 31-37]

The creative development of cybernetics has in a thoroughgoing and entirely convincing fashion corroborated the need for the most intimate contact between it and the philosophy of dialectical materialism. The solid connection disclosed between the pressing problems of philosophy and the problems on which cybernetics is working has demonstrated that this science is at the present time proving to be an exceptionally significant general-scientific methodological tool for investigating a multitude of natural, social, and spiritual processes. Such a tool is, in particular, made up of the cybernetic categories of 'information,' 'control,' 'organization,' 'self-organization,' 'feedback,' and other concepts, as well as the treatments of new fields of mathematics, which these categories are stimulating, and the very rich technical resources of cybernetics which are being applied in extensive fields of science and practice of ever-increasing range.

One of the cardinal philosophical problems in cybernetics has been and remains the problem of artificial "intelligence," the problem of whether a machine can think, about which the heated debates have not even yet died down. Some believe that this is not an urgent problem, others that its urgency is of the highest degree, still others that the problem is not a rightful one, a fourth group that the very attempt to model the human intellect is stupid, while even others in turn cherish a fifth opinion that the preceding one is unintelligent, and so on. This debate is going on both in oral communication and in the press. Thus, the newspaper Vecherniy Minsk of 29 December 1967 published an article by Professor A. Karlyuk entitled "Can a Machine Think?" This article enunciated the idea of the qualitative, sociohistorical specificity of man and his reason, of the ancillary role of "logic" machines, and of the fundamental irreducibility of human thinking to the logic operations of cybernetic devices.

In reply to this article the Zvyazda of 14 March 1968 ran an article by P. Protaseni and A. Rakov captioned "Not Advice, but Charades," which in a coarsely hard-selling and satirical tone spoke of the first

thus, "A. Karlyuk has taken up a strange position for a scientist. He believes it offensive and insulting to human dignity to compare the intellect with 'thoughtless automata.' This 'noble indignation' serves as the grounds for concluding that 'attempts to replace man by a machine must be refuted in advance.' But what do the facts confirm? The author's thesis contradicts the whole history of the development of culture, the most important feature of which is precisely the replacement of man by a machine, first in the area of physical, and then of intellectual labor."

In P. Protaseni and A. Rakov's article it is asserted that the problem of modeling the mental functions of man on a computer is "quite realistic and to a significant degree effected in practice."

The thought occurs to me that the different viewpoints in these arguments issue from the different understanding of such fundamental categories as 'man,' 'thinking,' 'consciousness,' 'machine,' 'image,' and a succession of others.

It must be stated that philosophy has no basis for erecting any methodological barriers either in cognitive or in creative activity. The history of science, particularly in recent decades, has entirely altered our concepts of the possible and the impossible. The thoughtful modern philosopher is inclined to have a more skeptical attitude toward scientific dogmas. He takes a different approach even to what previously some tended to regard as something impossible. Nowadays the term 'impossible' is becoming increasingly discredited -- if, of course, it is something within the framework of objective laws which is under discussion.

Dialectical materialism proceeds from the fact that every sort of pattern is concrete and qualitatively specific. Every level of the structure of matter must be approached with regard to its qualitative determinacy. In this the higher includes the lower as one of its premises and at the same time as its own basis.

When scrutinizing the problem of "consciousness and cybernetics" it must be taken into account that the problem of "man and his intelligence" is not only and not so much a cybernetical problem. This is a problem of the whole composite of modern sciences of nature, society, and man.

The controversy which is occurring over this matter is taking place because some authors are unrightfully claiming that the problem of human intelligence, its structure, and the feasibility of modeling it is entirely encompassed within the frameword of the categories which cybernetics utilizes. This is an unlawful claim. The human brain and the products of is activity represent an extraordinarily complicated phenomenon of physicochemical, biological, physiological, psychological, logical, linguistic, cybernetic, sociological, esthetic,

and philosophical nature. Threads of all the forms of motion of matter known to science have seemingly woven themselves into a single continuous unit in the brain. On the level of cybernetic modeling such cardinal characteristics of the consciousness as 'subjective image,' 'the ideal,' 'want,' 'motive,' 'conscience,' and so on remain beyond its limits.

One of the important problems in this sort of composite investigation of man and his intellect is to uncover the social determination and internal conditions of creativity in all spheres in which it manifests itself, as well as the structure of our intellect and consciousness.

In the approach to the problem of artificial thinking intolerable onesidedness of this type is sometimes perpetrated -- the supposition that thinking arises from intraorganic physiological brain processes, that it is a natural characteristic of the individual, and that brain thinks just because it is the brain. The wrong belief is encountered that everything depends on how the brain is organized and what physiological processes go on in it. Hence it is deduced that it is enough to create a model of the brain for this model to begin to produce thoughts, ideas, feelings, and efforts of will. This assumption is naive.

The gist of the matter is that intellect is not simply a natural property of the brain and of man. In the last analysis the true subject of consciousness and reason is not only not the brain, and not even man as such, but society. A social organism thinks in the person of man by means of his brain. Man thinks in a social fashion only as the subject of history. Man learns to think by mastering the logic of utilizing the pragmatic world which is undergoing creation by all preceding history, logic, and by all culture. Always and everywhere man carries with himself his whole individual history and the history of mankind. The hand and the brain, nourishment and multiplication, hereditary changes, even pathological changes are, in Marx's words, the result of past world history. The specifically human level of the determination of mental processes, processes of thought, consist in the social conditions of existence.

Man is a social creature representing the highest level of development of living organisms on earth, capable of producing the tools of labor and of employing them in his action on his environment, and possessing articulate speech, consciousness, and weltanschauung. Man is the subject of labor, thought, sensations, will, beliefs, and communication. He is the subject of scrutiny of the totality of social sciences, as well as of certain divisions of the natural sciences, medicine, and the technical sciences, which analyze man from their own specific angle of contemplation. Philosophy, expressing the essence of man and of his relation to the world, has been summoned to effect an integral theoretical investigation of man.

In pre-Marxist and non-Marxist studies such constituent characteristics of man as intellect, the ability to operate with symbols and to think, are usually distinguished. It is precisely herein, and often only herein, that the decisive distinction between man and animals is seen. Marxism has demonstrated that these properties do indeed comprise the characteristic features of man, but that they are derivative, not original, in nature in determining the essence of man. The initial characteristic of man is the capacity for effecting conscious transformation of reality by artificially created tools. "The first historical act...of individuals by which they differ from animals does not consist in their thinking, but in their beginning to produce the means which are necessary to them for life" (Karl Marx and Friedrich Engels, Sochineniya [Works], Vol 3, p 19).

Although consciousness and self-consciousness are essential to man, man is not identical either to consciousness or to self-consciousness, as idealists assume. As does any living creature, man need his bodily nature, for the body constitutes his only natural life. But man cannot be reduced either to his spiritual principle or to his bodily organization. He is a unity both of the natural and of the social, both of the physical and of the spiritual, both of the hereditary and of that developed in life.

Marxism refutes metaphysical and idealistic ideas of the existence of innate ideas and capacities, unchangeable properties of the psychic life of man. The anatomophysiological instincts with which a child is born cannot in themselves lead to the rise of complex psychic peculiarities. These traits do not appear in the process of the individual development of a person, but are formed, for the child learns to be a person in communication with adults. The assimilation of social, historically laid-down types and forms of activity and the transformation of them into his own active capabilities are the main condition and the decisive mechanism in the individual process of becoming a man.

The starting point of the Marxist concept of man is treatment of him as a derivative from society, as the result of rise and development of social-labor activity, for in his whole physical and spiritual being -- upright gait, cerebral structure, general facial features, the shape of his hands, his speech, emotions, and intellect -- man is indebted to the labor and social relationships which have taken shape on that foundation.

Man is not something once and forever given and completed. He is a concrete-historical being changing in the course of active transformation of the natural and social world. The formation of the physical construction and of the spiritual structure of man is the product of universal history. Man's intellect, his spiritual capacities and interests, are formed in labor and embodied in its results. By what signs, asked Lenin, can we judge of the real thoughts and emotions of the real man?

And he answered, "Naturally, there can be but one such sign -- action."

The natural prehistory of man preceded his social history. This prehistory was the evolution of the anatomophysiological structure, the germs of labor activity among the anthropoid apes, the development of gregarious relationships in the higher animals, and the evolution of audio and motor means of signaling. But the comparatively high level of development of animals in the anthropoid ape stage immediately preceding the appearance of man contained only the opportunity for the genesis of man. The deciding condition of the actual transformation of the anthropoid ape into man was labor. The start of manufacture of artificial tools of labor signified the start of the rise of man. Labor proved to be the determining influence on the development of consciousness, on the perfection of the cerebral structure and of its cognitive capacities. "First labor, and then and together with it articulate speech were the two most important stimuli under whose influence the brain of the apes was gradually transformed into the human brain" (Friedrich Engels, Dialektika prirody [Natural Dialectics], 1955, pp 135-136)

Animals cannot produce radical changes in the conditions of their existence; they adapt to their environment and depend on it; they are fettered to a certain element -- air, water, or the dry land -- which determines their mode of life. Man himself creates the conditions of his existence by transforming his natural environment. Here man differs from the animal "not only in that he changes the form of that which is given by nature; in that which is given by nature he simultaneously also realizes his own cognitive goal which as a law determines the manner and nature of his actions and to which he must subject his will" (Karl Marx, Kapital [Capital], Vol 1, p 185).

Before every person entering life is spread out the world of objects and social transformations in which is embodied and objectified the activity of preceding generations. It is exactly this humanified world in which every object and process is, as it were, charged with human meaning, social function, and purpose that surrounds man. And only through it does man enter into connection with nature. Assimilating this already humanified nature the child in various ways unites with the human essence and the existence of culture. In this union of the person each of his human relationships to the world -- sight, hearing, smell, taste, touch, thought, contemplation, emotion, desire, activity, love -- in a work, all the organs of his individuality, participate .

The historically forme standards of right, morality, domestic life, the rules of thinking and grammar, esthetic tastes, and so on from the very beginning form the behavior and reason of man and make of every individual man a representative of a certain mode of life and level of culture and psychology. "If by his nature man is a social

being, then it follows that only in society can he develop his true nature, and the strength of his nature must not be judged by the strength of isolated individuals, but by the strength of all of society" (Karl Marx and Friedrich Engels, Sochineniya [Works], Vol 2, p 146).

In opposition to individualistic teachings in which the individual man and his uniqueness come to the fore as the primordial given fact, as an identity enclosed in itself, Marxism regards man as something which has been conditioned by social relationships: every man bears all history with himself. The understanding of man as a social being is deeply based by Marx, who wrote that "the essence of man is not an abstract proper to an isolated individual. In its actuality it is the totality of all social relationships" (Karl Marx and Friedrich Engels, Sochineniya [Works], Vol 3, p 3). This thesis has enormous methodological significance, giving bearings for consideration of man as a socially determined creature, not as an isolated monad.

In criticizing the concepts that man is some isolated and self-enclosed monad Karl Marx emphasized that individuals "create each other" both physically and spiritually and the development of the individual is stipulated by the development of all the other individuals with whom he is in direct or indirect communication. The perception by man of himself as himself is always mediated by his attitudes toward other people. "In some respects man is reminiscent of a commodity. Since he is born without a mirror in his hands and not as a Fichtean philosopher ("I am I"), man first looks at himself, as in a mirror, only in another man. Only by relating to the man Paul as to one like himself does the man Peter begin to relate to himself as to a man" (Karl Marx, Kapital [Capital], Vol 1, p 59).

Man is included in every aspect in the context of communication with society, even when he remains alone with himself. Even, wrote Marx, "when I am engaged in scientific and like activity -- acitivity that only in rare cases I can perform in direct intercourse with others -- even then I am engaged in social activity because I am acting as a man. As a social product I am not only given material for my own activity -- even the language itself in which the thinker works -- but also my own existence is a social activity; and therefore even that which I do of my own person I do of myself for society, acknowledging myself a social being" (Karl Marx and Friedrich Engels, Iz rannikh proizvedeniy [From the Early Works], Moscow 1956, p 590).

Emphasis on the social nature of man does not mean ignoring his biological side -- either of the general or of the individual characteristics of his bodily organization. Every individual is a unique individuality in the whole make-up of his physical and spiritual traits and at the same time he bears in himself a universal human principle, a certain generic essence. He makes his appearance as a personality

when he attains self-consciousness, understanding of his social relations, and interpretation of himself as a subject of historic creation.

Man's natural individual characteristics also participate in the process of forming him, but they remain, as it were, neutral in respect to the substance of activity. The mental capacities and properties of man are formed during his life in society and are determined by concrete social conditions. Man raises himself to the level of personality by the force of the historic process, passing through the immense history of his development, beginning with the gregarious state, to the summit of modern culture. Under the conditions of life of the generic collective the individual man still does not become independent with respect to the community. Personal interests still have not been segregated from the interests of the collective, and the personality, as such, is still absent. During social differentiation and the development of personal rights and duties the individual more and more distinguishes himself from the collective and becomes a personality.

The naturalistic treatment of man and his intellect, a treatment powerless to explain his constructive-creative activity, is overcome by the thesis that the key to the understanding of man and of his intellect is in the hands of society which regenerates itself through daily objective-practical activity that transforms both the external world and man himself. The transforming action of the objective -- that is, the historical, not the organic -- reality also embraces the higher cognitive processes, thinking, and the initial sense forms, and the realm of emotion and will, in a word, all the elements of the structure of consciousness in which the principal role is played not only by the scientific, but also by the artistic method of reproducing reality. And in the artistic method of doing this cognition has coalesced with the function of creation and of spiritual-moral communication. Artistic cognition and creation are, in the words of Goethe, not for the world outside of man, but for the world which is in conformity with man.

Cognition in a certain sense is perception not only of surrounding reality, but also of one's own relationship to this reality, and not merely one's own relationship to this reality, but also of the significance of what is being done for society as a whole. Since man's activity has some particular social significance, the consciousness of this is characterized above all by the degree to which man is capable of realizing the social consequences of his activity. The greater the place occupied by manifestations of social duty in the motives of human activity, the higher the level of realization is.

The capacity of man for thinking is not directly included in the very structure of the brain; it is formed by the logic of objective-practical activity through uniting with historically amassed culture, through education and instruction, and through objective activity making use of procedures and means created by society. The richness of

man's inner world is a consequence of the richness and versatility of his social ties.

This is why effective modeling of the consciousness, its structure, and its functions cannot restrict itself merely to reproducing the structure of the brain. This requires, I emphasize, that the logic of the whole history of human thought be reproduced, as the individual entering life reproduces it. And this means repeating the whole path of man's development and supplying it with all needs, including also the ethical and esthetic requirements with their natural-biological premises and social content. Therefore Academician A.N. Kolmogorov is right in saying that an automaton capable of writing poems on the level of the great great poets cannot be constructed in any simpler way than by modeling the whole development of the cultural life of the society in which poets really develop.

The problem of man and his intellect is not so much a natural-scientific and cybernetic problem, as principally a profoundly social one.

Just as mathematical logic is unable by its own means to express completely, and even less so to explain the nature of the real process of human thinking (nor does it claim to do this) -- so cybernetics cannot claim to exhaust the essence of man, his intelligence, and his thinking. This demands involvement of the whole arsenal of modern research methods, and not only those at the disposal of cybernetics.

The above does not at all mean to deny the possibility of modeling thought. Such modeling is a present fact. Electronic computers are very successfully modeling the mechanism of formal logical reasoning proper to man. But this mechanism is far from exhausting the developed consciousness of modern social man, the "flexibility" of thought, and its effectiveness in solving the most diverse problems -- an effectiveness that is not stipulated by any previously laid-down rigid system of formal rules. This mechanism of creative thought has still been subjected to extremely little research, but it is obvious that it is somehow linked to the capacity of operating not with rigidly determinate and at the same time semantically capacious ideas, but with "vague" ideas, with sensual and intellectual intuition, and with the capacity of effecting extensive and pithy analogies and hypotheses based on a gigantic store of scientific facts, observations, and ideas won by the whole history of mankind.

Experience has shown that it is relatively easy to model some comparatively narrowly specialized types of cerebral work, for example, the performance of calculating operations done by a worker in some department of a bank. But such modeling does not encompass the most general and the most important mechanisms of the brain's activity. The human brain is universally capable of solving assignments of an extensive class of problems, and at the same time it can accomplish

the specific individual procedures and methods of solving different problems of the most diverse degree of complexity and type characteristics. The important fact must be borne in mind that every person carries out trains of thought proper to him alone and often unique, as well as general trains of thought, when solving a particular practical or theoretical problem.

Misunderstanding often arises in connection with the different concepts of the essence of the machine. Here is a typical definition: in cybernetics machine is the name given to a system capable of performing acts leading to a certain goal. Hence living beings also, and man in particular, are machines in this sense. The goal is interpreted as a state of the system, determined by a natural process or by human efforts, toward which this system is regularly tending, without in the process having any conscious intention.

But in such a motion of the system -- here I use the term 'motion' in its philosophical sense -- of, for example, a logic machine, there is no goal in the genuine sense of the word. This term is employed here in an expansive fashion. In philosophy the goal is a human want which is idealized and has found its object, a subjective image of an object of activity, in whose ideal form the result of the activity is anticipated. A goal ensues from the realization of a want in some object and has no existence outside of wants.

Want, indeed, is the cardinal criterion of everything alive. Moreover goals are formed on the basis of the complete totality of mankind's experience and are raised to the highest forms of their manifestation in the form of social and esthetic, moral and scientific ideals (say, the creation of a society in which the happiness of some will not be built on the unhappiness of others, and so on).

And here we again come up against the profoundly social nature of man, his activity, and his reason.

Can cybernetics approach man and his intelligence as it would a machine? Yes, it can. It does this on the same basis that the physiology of the higher nervous system, when investigating the machinery of consciousness, divorces itself from the intension of consciousness, from the essence of consciousness and thinking itself. In exactly the same way cybernetics does not study either man, or thinking, or creation in the proper sense of the word; cybernetics uses the result of the research on these phenomena by the whole composite of sciences -- philosophy, psychology, the physiology of the higher nervous system, and others and strives with the aid of automata to imitate certain aspects of the operation of the brain.

The essence of the machine as such was defined long ago by Marx. This definition is true also for the present day. "The machine is

natural material converted into organs of the human will and of its active manifestation in nature. They are organs of the human brain which are created by human hand, the objectified force of knowledge."

This definition is so general -- and at the same time so precise -- that it refers to the machine at any level of its perfection, inclusively also to cybernetic automata, the goal of whose creation is to liberate man from the labor which may be entrusted to machines -- the "amplifiers of the intellect."

The goal of models of thought activity is not, of course, to create thinkers, poets, writers, political and state figures -- people in general -- or to replace them, but to employ cybernetic methods for technical progress, as well as to move forward in the understanding of the essence of thinking itself, of consciousness.

The fact must be stressed that if previously both the nature of thought and the nature of the human consciousness seemed rather clear, then the demands of the development of cybernetics and of technical progress are forcing both psychologists and philosophers to take deeper thought on the nature of human consciousness and thinking. They are, in particular, forced to submit the nature of the mind to analysis anew, as well as that most rich realm of feeling, without which not a single thought is engendered in man's head.

The development of science and practice will indubitably get rid of the now existing and equally groundless extremes -- the speculatively dogmatic, skeptical approach to the potentialities of cybernetics associated with lack of faith that certain logic operations considered the privilege of man alone can be reproduced in machines, as well as the sensational-advertising approach to cybernetics expressed in the attempt to identify the machine with man, intellect, and the imitation of some of its properties. P. Protaseni and A. Rakov's article, "Not Advice, but Charades," can serve as an example of the second extreme. Neither of these extremes is of any help to the actual progress of cybernetics.

In a few years cybernetics has achieved substantial results, both theoretical and practical. And it has no need of sensational claims like the replacement of man with "creatively thinking" machines possessing consciousness and self-consciousness -- claims which strive for outward effect.

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READING AUTOMATION

[Article by A. Zelentsov; Moscow, Nauka i Zhizn', Russian, February 1969, pp 30-32]

People became bored ages ago with the reading of some texts. It is uninteresting to read, say, some report on warehouse supplies, or on the output of some component or another. But they must be read to generalize information and compile other summaries, tables and reports. Computations of this type are now being handled by computers, but the initial data must be fed into these machines by means of punched cards. However, punching takes up too much time and manual labor, sometimes completely canceling out the effect of using computers. This is always the case when the computation itself is simple and there are many initial data.

To solve problems of this kind, reading machines are needed which allow computer input of documents without the use of punched cards, reading these documents automatically.

It is true that contemporary reading machines are a long way from being able to read a handwritten text. We must be content with machines which can read letters and numbers machine-printed in a special type style. But even in this case, the machines can find application in the most diversified fields.

It would seem to make no difference whether the text is retyped or keypunched into a card. But there is a difference.

In the field of economy, there is the concept of the "primary document." And if this primary document is machine-printed, then a reading machine may be quite advantageous since the primary document itself may be fed into the computer without making a copy and thus destroying the legality of the document. Therefore to compute the payroll at a plant where 30,000 workers are employed directly from the time cards, or to plan the supply of materials from various statements, not to mention banking and postal operations, it is very convenient to use machines which read letters and figures.

A reading machine for machine-printed letters and figures is being developed at the Institute of Cybernetics of the Academy of Sciences of the

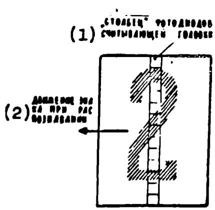
Ukrainian SSR. This machine is called "ChARS [Chitayushchiy Avtomat s Registrom Sdviga; Reading Machine with Shift Register]. The basic distinguishing feature of "ChARS" is in the words "shift register." The unusual use of a shift register allows the machine to read 52 symbols in ordinary, rather than stylized, machine-printed type.

Vladimir Antonovich Kovalevskiy, director of one of the departments of the Institute of Cybernetics, says:

"We haven't turned away from stylized type and special machines; this is a completely separate project, which means that it's extra work. If we are counting on large-scale use of reading machines, then we must set our sights on widely used printers."

The operating principle of the "ChARS" consists in comparing the image of a letter or figure with master patterns. Naturally this method limits the sphere of application of the machine somewhat since the "ChARS" can read only one type face with a given font of masters, e. g. the type style of the "Optima" typewriter. But the master font may be replaced by another so that the machine may be converted to read the type face of the "Moskva" typewriter for instance.

The comparison is made by computing what are called correlation coefficients, i. e. quantities which indicate the degree of similarity between image and master. This method in itself is nothing new, it is used in a number of other reading machines as well, but the symbol is compared with the master in the "ChARS" not once, but eighty times -- ten shifts



A magnified image of the symbol in the field of view of the reading head: 1--"Column" of photodiodes in the reading head; 2--Motion of the symbol during recognition.

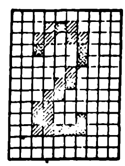


Image of the symbol entered in the shift register.

along the horizontal, and for each of these horizontal shifts, eight shifts along the vertical. And it is this that constitutes the special use of the shift register. We have selected this method to achieve maximum readout reliability. The fact is that regardless of the care taken in filling out the documents, it is nevertheless impossible to wind up with the symbols on the documents arranged in a strictly defined way with respect to the optical system of the reading machine. Some displacements are always unavoidable, and it is therefore difficult to line up the symbol and the

master. In most reading machines, this problem is solved by what is called centering with respect to the edge. In other words, the left edge of a symbol is first located, then the upper edge, and then the image of the symbol is shifted so that these edges line up with a given area in the master pattern field. The reliability of this method is low since the edges of the symbols are just the areas which are most liable to be distorted in various ways. This is just the area where there is the greatest likelihood of smudges or breaks. Instead of this, the "ChARS" performs what is called examination by shifts, i. e. the symbol is barely shifted and the correlation quantity is determined and so on — 80 times. When this has been done, we are sure to have found the position in which the symbol best coincides with its master. In this case we get a correlation maximum. If the given maximum corresponds to master pattern "m" it means that the letter "m" is in the machine's field of view; if the maximum is that for master pattern "7" then the machine sees a figure 7.

In addition to everything else, the "ChARS" has still another advantage: it can distinguish symbols in a line even in the case where there are no spaces between them. After all, symbols printed by an ordinary typewriter frequently touch at the edges. Therefore reading machines which distinguish symbols by spaces can not read an ordinary typewritten text.

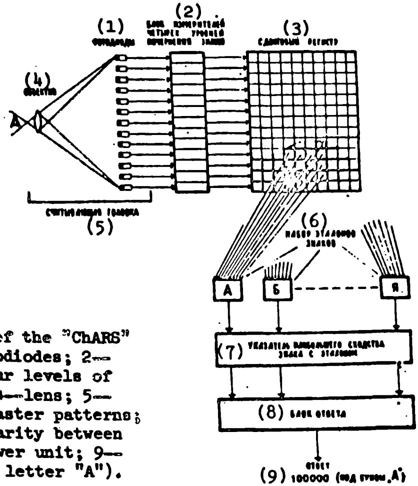
Symbols are distinguished in the ChARS by means of all these same correlation maxima. To distinguish one letter from another, the "ChARS" does not look for a space, but rather after noticing one maximum in similarity waits for several horizontal shifts to see if a second maximum appears which is greater than the preceding one. If no such maximum appears after a time sufficient to pass over a letter, the "ChARS" takes the preceding estimate as final.

We have turned all our efforts to achieving maximum readout reliability. After all, if the "ChARS" makes even one error in a thousand cases, it can never be used on jobs, let us say, in banks. Therefore in developing each unit we have tried to do everything possible to achieve maximum utilization of the information contained in the image of a letter. And for this reason the "ChARS" now distinguishes not merely black and white, but four shades of gray as well. Imagine that some line is faint because the key has not been struck hard enough, or the ink ribbon was worn, or because of a bad spot. The "ChARS" never skips a faint line and never confuses the real line of a letter or number with a spot because the device has a sufficiently high gray resolution.

Let us now consider how all this takes place in practice.

A narrow metal chassis crammed with rows of Pertinax plates to which electronic components are fastened, a bundle of wires strung overhead connecting the chassis to a complicated mechanism standing on an ordinary wooden table, and on and under the table all kinds of instruments with wires running to the mechanism. This is how we saw the "ChARS."

"All this will look completely different in the first model which is now being made at an experimental plant," explained Vladimir Antonovich. "All that you see in this room will be concealed in two compact cabinets. The reading mechanism will be located under a transparent plastic cover and will look completely modern. A stack of documents is placed in the reading



Simplified block diagram of the "ChARS" reading machine: 1—photodiodes; 2—unit for measuring the four levels of gray; 3—shift register; 4—lens; 5—readout head; 6—set of master patterns; 7—index of maximum similarity between symbol and master; 8—answer unit; 9—response 100000 (code for letter "A").

mechanism. They are all the size of a standard typing sheet which accommodates about forty lines with fifty to seventy symbols each. Pneumatic suction devices take the top document and feed it to the so-called transport mechanism."

Vladimir Antonovich stepped a little to one side and turned on... a vacuum sweeper. A sheet of paper passed to a rotating drum and wrapped itself around it.

"We used a vacuum sweeper to solve our problem," laughed Kovalevskiy, "only in this case the air is sucked from a drum in which small holes are drilled. That's why the sheet of paper sticks to the drum."

The reading head moves along the drum. With a single rotation of the drum on its axis, a single line has passed the reading head. The head is then shifted to the next line and another rotation takes place. In this way the entire sheet is read and replaced by the second.

The average speed of the "ChARS" is 200 symbols per second. The average speed is that which can be attained in reading actual documents with losses of time in searching for lines, changing documents, etc. And if the symbols could be printed right up against each other like a kind of chain without spaces, the "ChARS" would be able to read 540 symbols per second.

Now a few words on the sequence of operations.

The reading head is an optical system with a source of illumination which projects a magnified image of the letter onto a photodiode "straightedge." The photodiodes are miniature photocells which react to light and produce an electric current proportional to brightness. All 18 photodiodes sense the part of the letter image which has the form of a narrow vertical column. The electrical signals from the photodiodes are recorded in a

single column of the shift register. As the drum rotates, an image of an entire letter is built up from ten successively recorded columns in the register, and so on, symbol after symbol. The electric signals are then sent to the shift register — the largest part of the reading machine. Connected to the shift register are the master patterns. These masters are sets of resistors. There are about 5,000 of them. The magnitude of the electric current which appears at the output of each of them is a measure of the similarity of an image to the corresponding master pattern. All this takes place automatically.

After the maximum current has been found, i. e. after it has been established that an "a" is an "a" — this information is sent to the computer memory for further processing.

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REFLECTIONS ON THE STRUCTURE OF INDUCTIVE LOGIC

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1. The Nature of Inductive Conclusions

In comparison with the theory of deduction which has been elaborated in modern logic it seems that inductive logic is an imperfect and rather artificial structure. Some authors entirely doubt the very possibility of creating a rigorous logical theory of induction. At the same time inductive logic has a long history of its own, no less long than that of deductive logic. It was precisely the elements of inductive logic that were contained in the Canon or On Logic, assumed to be by Democritus, apparently the first treatise on logic in Ancient Greece, but one which has not come down to us. Since that time the problems in drawing inductive conclusions has occupied and continues to occupy philosophers, logicians, and methodologists of science. What, then, is the reason for the very modest progress of inductive logic if minds no less profound and penetrating have been engaged in it than those which laid the foundation and erected the lofty building of modern deductive logic systems thereon?

The alluring simplicity and rigor of many of the constructions of deductive logic have in the last analysis proven to be based on the extremely forceful abstraction of the absolute truth of the statements figuring in this logic. Deductive conclusions amass within themselves only the information which is contained in the premises of the deduction, and the apparatus of deductive logic can comparatively easily be so structured as to separate these conclusions from their premises as independently true theses by transferring the characteristics of truthfulness from the premises to the conclusions.

In the same logical constructions in which this abstraction does not occur because of the point of the matter we as a rule have no purely logical means to separate conclusion and premises in this fashion. And even the employment of extralogical means of various sorts for this separation cannot eliminate the indeterminacy of the

of the meaning of the conclusions derived in these constructions. It is this very thing which occurs in inductive conclusions, where the imperfection of experience, which generally shows up as one of the basic factors in the relativity of human knowledge, is manifested both in the incompleteness of the analysis of the information contained in their empirical premises and in the claims of the conclusions in these inferences to greater information than was evinced in the premises. And, although one of the cardinal assignments of inductive logic is as much as possible to eliminate the conclusional indeterminacy which therefore occurs, or at least to appraise the degree of it, nevertheless complete achievement of these aims eludes us and the stamp of indeterminacy always is borne by conclusions from induction.

The attention of modern investigators of inductive logic is being concentrated exactly on sizing up and developing methods to permit the fastest possible full and profound analysis of the information contained in the premises of inductive inference in order to make this conclusion as determinate and separable from the premises as possible. Thus, the famous inductive conclusion "All swans are white" had a determinate meaning only with respect to the limited experience on which it was made. From this aspect the discovery of the black swan in Australia merely added to the preceding premises still another, which, to be sure, struck out that conclusion itself, but only that one. Based on the old experience a certain conclusion had been made; based on the new and fuller experience it proved to be groundless. In itself this fact, so usual in human cognition, could hardly cause a sensation. It is another thing if the conclusion "All swans are white" emerges as a thesis entirely separate from its premises and moreover the separation of this conclusion from the premises is achieved by certain logical means. Then the discovery of the black swan subverts these means themselves, and we are now dealing not so much with the appearance of a new private theory as with a "crisis" in a definite logical conception.

Viewed from modern inductive logic this situation is explained by the lack in traditional translational induction of adequate means to analyze the information contained in its premises -- lack of means to evaluate, and hence to a certain degree to eliminate, the indeterminacy of such a conclusion. It may in particular be demonstrated that if the inductive conclusion "All swans are white" is analyzed by several means of information analysis elaborated in modern inductive logic, then even before any discovery of a black swan it would be clear that the indeterminacy of this conclusion is too great for it to be made. The methods of this sort of analysis may differ in diverse theoretical models of induction. Thus, if the indeterminacy of an induction conclusion is thought to be its lack of authenticity, and the latter is estimated by a "probability" function introduced by certain special rules, which vary for the different systems of induction, then the result will be that the probability of the conclusion "All swans are white," even when made on the basis of experience containing no contradictory example, is extremely small.

Here, for example, is how Reichenbach performs this analysis, using his theory of different levels of frequency [1]. He constructs several "storeys" of induction, that is, regards induction not as only one sequence of observations of swans, but as a number of sequences about different genera of wildfowl. In the thus-obtained table the horizontal lines are made up of observations on different specimens of one and the same genus of bird and characterize induction by specimens while the vertical columns are now composed of observations on specimens of different genera and characterize induction by genera. In this case induction by specimens may be evaluated by the first-order probability (wager) which is described by the relative frequency of specimens of this genus which possess a certain distinctive feature (color in our case). Induction by genera, however, may be evaluated by the second-order probability (wager) which is characterized by the relative frequency of the genera possessing some distinctive feature among all the genera investigated (in our example color is a constant characteristic within the genus). And then it becomes clear that color is not at all any constant characteristic within the biological genus of birds because the second-order probability here is extremely small. And although the empirically established relative frequency for individual sequences with respect to specimens of the genus may indeed be unity (as was the case with the statement that all swans were white which was made before the discovery of Australia), the probability itself of the truth of this conclusion is insignificant. Here the inference of induction through listing applied to one sequence is corrected by another induction which now regards every sequence as an element.

This example also shows in particular the groundlessness of the opposition between enumerative induction and ampliative inference which was so insistently emphasized in the traditional inductive logic of Bacon and Mill. Correctly conducted enumerative induction is threatened by a "crisis" in logical means of inference to no greater degree than is the logic of ampliative inference, which obviously contains logical means of analyzing its premises and checking the validity of the conclusion. But some logical means for this analysis are adequate in neither type of induction, and still other different sorts of extralogical means are employed to effect the most determinate and strictest possible inference. In the Reichenbach analysis quoted this extralogical means is a certain substantial knowledge of the biological classes of birds and their species which in the last analysis determines the limitation which makes this principle of induction effective.

But if induction is ineffectual in its purely logical principle can inductive logic be regarded at all as a logical theory proper? A negative opinion on this score could be consistently justified from the positions of extreme logical formalism and nominalism. Meanwhile the effective interrelation between logic and specific sciences in general and logical and extralogical means in particular is outlined for us on another plane from the one on which it presented in the concepts just

mentioned. Generally speaking, any logic system (in contrast to a properly logical enumeration) includes not only syntaxis and deduction, but also semantics and pragmatics. The latter are associated right with the features of a particular application of logic. And if it proves inadequate to effect the construction of a logical system of purely logical means, then, generally speaking, it is permissible to enlist even nonlogical means. This, by the way, holds true not only with respect to inductive logic, but also with respect to a number of deductive logical systems, for example, systems of metrical, in particular, probabilistic logics.

The thus constructed systems must be considered just as logic systems since other systems -- theories -- are built on top of them, as on a logical basis. The is particularly distinctly displayed in the case of constructing theories on the basis of formalized languages. In addition to the logical part a formalized language of theory also contains a certain specific part -- a specialized language making it possible to describe precisely the concrete, substantial region whose description this system is. As such a specialized language a formalized language may contain the axioms of arithmetic, of classical or quantum mechanics, and so on. If, however, the language of theory does not contain a formalized logical part it is not a formalized language at all, but merely a specialized one.

Thus, the language of quantum mechanics, although including a solid mathematical apparatus, will not be a properly formalized language until its logical part is formalized, that is, those logical means which are utilized in reasoning in this science. At the same time these logical means themselves somehow depend on the special language of the given science. To use A. Church's expression, it may be said that the formalized language corresponds to the logical form of the reasoning carried on in the given special language. The formalized language of theory therefore must contain all the logical means for effecting the necessary reasoning in the given special language. Then the logical part of the formalized language of theory may with complete justification be called the logic of this theory, and in cases where the special languages of theory are sufficiently rich the logical part of the formalized language may organically include even extralogical means. This is precisely the situation in inductive logic, which deals with a far richer world than does deductive logic and consequently must also possess richer means to describe it.

Any logic understood to be an interpreted system, that is, a system with a clearly prescribed semantics, has certain ontological premises and is essentially constructed as the logical model of some ontological system. Thus, Birkhoff and Neumann note that classical logic is not suited to description of the microworld since simultaneous observability of a number of characteristics of objects is not satisfied, and this makes the law of distributivity unfulfillable in

its logic model [2]. Essentially different ontological systems, generally speaking, also require different logical models. This is true both for deductive and for inductive logics. Therefore on condition of sufficiently specific and delicate analysis we will have not one, but a multitude of systems of inductive logic, each of which will be the specific logic model of a certain sort of ontological system.

In the actual construction of different inductive logics this is realized in the variability of the extralogical postuales which are added to some original logic base, or even in the variability of these bases themselves. We would note that with such an understanding of inductive logic even the so-called problem of justification of induction, as it has been traditionally formulated in philosophy since the time of Hume, is essentially eliminated, and moreover eliminated in a more natural way than by Reichenbach, who for this purpose actually resorts to the abstraction of potential realizability.

It must moreover by borne in mind that these terms themselves -- "logical" and "extralogical" means -- are relative and correlative in nature. Means which are extralogical in one context may make their appearance as logical means in another, and conversely. If certain means in the construction of any theory belong to it specialized language, then they will be extralogical means with respect to this theory. If, however, in the construction of the theory they belong to its logical language, then they will now appear as logical means with respect to this theory. Thus, certain substantial induction premises are "extralogical" means with respect to the classic means of deductive logic. And these same means may be attributed the "status" of logical as soon as we are dealing, for example, with cases of inductive foundation of particular knowledge. The captious critic of the point of view expounded above should bear in mind that the term "extralogical means" which we previously used must be corrected in precisely this correlative sense.

2. Probabilistic Models of Induction

The founders of inductive logic -- Francis Bacon, Herschel, and Mill, as well as some subsequent logicians contemporaries of ours -- John Venn, Grenevskiy, and Rescher -- endeavor to construct induction as a rigorous conclusion like a deductive one. And if they even accept an indeterminacy conclusion in it, they nevertheless do not apply the function of "probability" to the appraisal of this indeterminacy. Other investigators, however -- Laplace, Keynes, Lindenbaum-Hosiasson, Carnap, Kemeny, Reichenbach -- approach the solution of the problem of induction from the standpoint of probabilistic evaluation of this indeterminacy. Here probability is interpreted as a certain logic characteristic, however it is measured -- it be the "degree of similitude" (Laplace), "degree of confidence" (Keynes), "degree of confirmation" (Carnap), or even "logical frequency" (Reichenbach).

One of the first probabilistic models of induction is that of Laplace. He uses the language of the mathematical theory of probability to analyze enumerative induction. This induction, as is well known, is defined as follows: if a certain number $\underline{n}$ of cases of class α be given which prove to be members of class β, and if moreover not one α is known which would not be a β, then the statements "the next case α will be a β" and "all cases α are β" both have a certain probability which becomes increasingly large as the number of cases examined is enlarged. Laplace proposes this sort of model for it: if there are N + 1 identical urns each of which contains N black and white balls and all possible combinations of black and white balls are found in the urns, and if then $\underline{n}$ balls are selected from an urn taken at random and they prove to be white, it is then asked: (1) what is the probability that the next ball taken from this urn will be white and (2) what is the probability that all the balls in the urn will be white.

Laplace's model has a rather distinct two-stage structure [3]. In the first stage Bayes' theorem, simplified by several supplementary assumptions, is used to determine the probability of the possible cause of a series of events known to us (the drawing of the white balls). In the second stage, now with this cause as the starting point, a search is made for the probability of future events in the same series. In his construction Laplace employs not only the principles and theorems of the theory of probabilities, but also assumptions from outside of probability theory. In determining the probability of a possible cause in his model Laplace introduces the assumption of the equiprobability of all the causes (urns), an assumption which cannot be derived from the data directly connected with our experience, but is established from more extensive data, most often from certain apriori considerations. Moreover in defining the probability of future events Laplace starts from the assumption that their cause will be the same (that is, will have the same probability as for the past events).

Having the first Laplacian assumption in mind Bertrand Russell properly calls it absurd [4]. It does, in fact, postulate an extremely powerful idealization which is very far from the real conditions under which enumerative induction is ever used. On the other hand, this sort of assumption is the simplest for making calculations and in general makes it possible to construct a graceful model of induction. It is interesting to note that Boole and Edgeworth's attempt to replace the Laplacian assumption of the equality of the apriori probabilities with a more complex one leads to extreme complication of the model, right up to the practical impossibility of making use of it.

As for Laplace's second assumption, in his model it obvious follows from the fact that all the experiments are conducted with a single urn, but it contains in hidden form the profound principle which underlies all inductive conclusions in general and which Mill formulates as the "principle of the uniformity of the structure of nature."

For comparison with Laplace's model we shall adduce one of John Maynard Keynes's models of induction [5]. In this model Keynes strives to determine the probability P_n of the inductive extension of g observed cases $x_1, x_2, \ldots, x_n$ in the context of some system of knowledge h -- $P_n = P(g/h \wedge x_1 \wedge x_2 \wedge \ldots \wedge x_n)$. Here some substantial apriori consideration $P_0 = P(g/h)$ -- a characteristic of the inductive extension made under the conditions of knowledge system h -- is drawn upon in addition to probabilistic and logic considerations. The relationship derived by Keynes has the following form:

$$P_h = \frac{P_0}{P_0 + (1 - P_0)\cdot P(x_1 \wedge x_2 \wedge \ldots \wedge x_n/\bar{g} \wedge h)}.$$

We see that $P_n = 1$ if $P_0 = 1$, and this occurs in the case where inductive generalization g ensues from existing knowledge system h or if $P(x_1 \wedge x_2 \wedge \ldots \wedge x_n/\bar{g} \wedge h)$ 0, that is, when in the context of the given knowledge h the cases $x_1, x_2, \ldots, x_n$ observed are incompatible and deny their inductive extension g. If $P_0 = 0$, then $P_n = 0$ -- and this occurs when inductive generalization g is logically inconsistent with the extant sum of knowlege h, and so on. Of the greatest interest, however, is the set of intermediate values of P_n in which must be computed the different relationships of the rate of approach to 0 or 1 by $P(x_1 \wedge x_2 \wedge \ldots \wedge x_n/\bar{g} \wedge h)$ when P_0 is fixed. Here satisfactory results may be obtained with very approximate calculations. Therefore the Keynesian interpretation of probability as a "degree of reasonable confidence" proves to operative in these cases. At the same the fundamental difficulties in Keynes's model are associated with establishing the exact value of P_0.

In comparison with Laplace's model that of Keynes obviously still somehow minimizes the extralogical inductive postulate which, generally speaking, is inherent in any form of inductive conclusion. Instead of the set of apriori possibilities in the Laplacian model we are here dealing essentially with one such probability. It is true that the main difficulty in both the Laplacian and the Keynesian system is to ascertain the exact numerical value of the apriori probability, but in the Keynesian model this difficulty is shifted to a somewhat different plane -- its solution involves a lesser number of premises, is of less formal nature, and obviously presupposes a more substantial logico-methodological basis.

It is our view that this minimization is mainly achieved by means of the new approach to the problem of induction. Induction, according to Keynes, ceases to be predictive in the sense of forecasting new observable cases. It deals only with the relationship of the hypothetical generalization to the observable data on which this generalization is built. It is essentially with Keynes that the new (somewhat onesided, we think) and presently predominating direction in inductive logic begins,

and whose characteristic features are: (1) application of the formalized language of probabilistic logic to analysis of induction, and (2) deductivism in the approach to the problem of induction -- transformation of this problem into one of corroboration (verification or falsification) of hypotheses.

In the two induction models which we have cited (as, generally speaking, in other also) the task of determining the apriori possibilities figuring in these models -- besides the action of logical principles -- is likewise perceived. It is this very point which is the most vulnerable and moot point in inductive models. Some believe that a certain extralogical postulate must be utilized to determine the apriori probabilities in inductive logic models. For Laplace, for example, this postulate was the principle of insufficient reason. Keynes argued sharply with Laplace who had attempted to apply the principle of inverse probabilities as the sole method of constructing probabilistic models of induction, and he did so just because this route leads to the need to use precisely this extralogical postulate in every specific inductive conclusion.

Keynes's conception is essentially that in every concrete sort of inductive inference for finding apriori probabilities one must be guided by practical considerations, that is, those making sense in regard to the case in hand, as well as by considerations of analogy. The problem of the fundamental extralogical postulate (or postulates) comes up in Keynes on another level -- that of the problem of basing or justifying inductive logic. As such postulates Keynes puts forward the principle of restriction of independent diversity and a more precise principle of insufficient reason, which he calls the indifference principle.

The Keynesian principle of restriction of independent diversity postulates that in the whole multifariousness of the facts or properties under investigation a certain restricted set of elemental, that is, independent, constituents may be segregated and that from the combinations of these constituents all the properties of this multifariousness may be composed. The equiprobability of these constituents is moreover postulated by the indifference principle. These two postulates thus underlie the fundamental possibility of computing the ultimate apriori probabilities and thus the feasibility of utilizing the theory of logical probabilities in inductive logic.

It is in this very approach that that which was new and Keynes's contribution to the theory of inductive logic made itself felt. It is our opinion that hitherto the criticism reproaching Keynes for these principles of his being completely inadequate for practical calculation of apriori probabilities have still underestimated this. Such reproaches are unjust because the aforementioned Keynesian postulates play an entirely different role in his theory of induction -- the role, so to say, of the axioms of existence of ultimate apriori probabilities, and by no means that of prescriptive rules for their practical computation.

It is interesting to note that the gist of the principle of restriction of independent diversity is somehow implicitly assumed also in the construction of Mill's induction since, for example, in every specific case of its application we have (and can have) only a limited set of concomitant circumstances under investigation. Mill's adoption of the concept of multiple causality has, in general terms, already created the prerequisite for the opportunity to apply the probability concept in his logic. The explicit formulation of the principle of restriction of independent diversity, along with the introduction of the principle of insufficient reason formulated in some particular manner (neither formulation made in Mill) would lead to the construction of a probabilistic model of Mill's inductive logic, including his methods. As is known, however, the idea of a probabilistic approach to induction was foreign to Mill himself. The principle of the uniformity of the structure of nature which he formulated -- rather philosophical than logical in its nature which made it possible to speak in general of the existence of persistent (that is, repetitive) causal connections -- has another significance and meaning in basing Mill's induction than the Keynesian principle of the restriction of independent diversity [6].

The Keynesian approach to the construction of induction on the basis of probabilistic logic was essentially the first properly logical and systematic approach which exerted a perceptible effect on all succeeding probabilistic theories of induction. We find consistent fidelity to Keynes's ideas and system in, for example, the works of Lindenbaum-Hosiasson [7]. Keynes's conception is developed on the basis of a more precise definition and more detailed elaboration of its basic concepts and principles in the inductive logics of R. Carnap and J. Kemeny.

Thus, Carnap, proceeding from the Keynesian principle of the restriction of independent diversity, introduces the new concept of "description of the state" as fundamental to his theory of induction [8]. This enables Carnap to formalize inductive logic more rigorously, it is true, because of the rather "rigid" formalization also of the language in which may be described the world in which the principles of his induction prove to be effective. This is a rather poor language -- its type is that of narrow calculation of one-place predicates with equivalence -- and thus permitting induction to be applied only to the world of characteristics, not of relationships. In Carnap induction there is continued retention of the significance of the Keynesian principle of the restriction of independent diversity, which is manifested here in the requirements of finiteness, as well as -- and this is particularly important -- of the independence of the elemental propositions comprising the base logic. Under the conditions of the formalization of induction performed by Carnap the requirement for this independence of propostions considerably impoverishes and limits the opportunities for application of induction. Bar-Hillel and Kemeny, for example, have repeatedly pointed out this fact [9].

In his theory of induction Kemeny travels the very path of watering down the Carnap demand for independence of the elemental assumptions. Kemeny achieves this attenuation by adding several supplementary axioms to the Carnap inductive logic axioms, and these added axioms change the universe of the "description of states" itself, enabling it in the last analysis to widen the opportunities for applying induction. Kemeny induction is therefore capable of embracing not only the world of characteristics, but also the world of at least binary relations.

The perceptible influence of Keynes may also be detected in the inductive logic of H. Reichenbach [1], but where Karnap solves the problem of establishing a basis for induction by more precise definition of the Keynesian principle of indifference and restriction of independent diversity by introducing them directly into logic itself so that his inductive logic is constructed as a certain integral system in which logical and extralogical postulates are no longer distinguished, Reichenbach on the other hand develops another idea contained in Keynes concept. And that is his idea that every specific inductive conclusion should be based not any special principles or postulates, but on practical considerations.

In the development of this idea Reichenbach goes considerably farther than Keynes in his belief that even the whole of inductive logic needs no sort of postulates to base itself, but is a "self-justifying" theory as soon as practical rational considerations have been found which are in conformity with the essence of the relationships under investigation. As H. Putnam expresses it, the inductive logic of Reichenbach is constructed like an optimum strategy, that is, a strategy which, although it guarantees no success in itself, nevertheless leads to success better than any other strategy [10].

Essentially, of course, neither is Carnap able to do without any practical prerequisites of substantial content in the construction of his inductive logic. And for his part Reichenbach cannot help using considerations of a theoretical nature. Thus, in order definitively to construct his <u>c</u>-function Carnap must resort to selecting a specific inductive method from the continuum of possible inductive methods, and this selection can be accomplished only on the basis of entirely substantial considerations [8]. Reichenbach on the other hand certainly has no poorer an understanding than his critics that to derive a satisfactory evaluation of a wager of the first order merely by increasing the number and scope of choices would be too vulnerable a spot in his theory. It is just in order to get rid of this defect that he constructs his "many-storied induction" in which the induction of a lower order is corrected by the inductions of higher orders.

In this idea of Reichenbach's it is not hard to detect an ordinary statistical approach to study of empirical material. In fact, in order to evaluate any property in a selection from some totality it is needful

in statistics to have a conception of the law of distribution of this property in the whole totality. A conception or assumption of this sort is properly also a second-order wager in Reichenbachs induction. This wager itself is not evaluated here, but only by means of it can the second-order wager be evaluated, that is, made qualified. Of course, should we want to evaluate or qualify a second-order wager, we need an unqualified wager of the third order, and so on. Reichenbach himself assesses this sort of higher-order wagers only as "practical" considerations. It is clear, however, that each such higher wager is in actuality based on experiences of an increasingly extensive class which also include theoretical considerations proper. In actual fact it is therefore found that the difference between extralogical postulates and practical considerations is erased even for wagers of the second order. That is why in Reichenbach every concrete inductive conclusion is based not on just one, but a set of extralogical postulates.

3. Other Approaches to Analysis of Induction

Analysis of induction with the means of probabilistic logic is not the sole method of analysis. The logical content of inductive problems has sides and aspects of its own which presume the possibility and the expediency of other approaches -- other problems which may be posed and solutions of them by other methods.

One such approach involves the application of so-called indeterminacy logic to analysis of inductive reasoning. This logic embraces statements which admit or combine several different values of truth. For example, in the case of a double-valued logic these statements can combine the value of "true" and "false," and in the case of an n-valued logic may combine m different truth values (where m is equal to or less than n).

Such reasonings frequently occurring in the practice of scientific (and particularly heuristic) thinking may serve as substantial motives and justification for the construction and investigation of indeterminacy logics as those in which premises with definite values of truth and conclusions indeterminate in their truth value play a part. We also meet up with just such a situation in the general case of inductive inferences where the induction result, generally speaking, is still unknown and the inductive conclusion may be either true or false. Attempts to model such inductive conclusions by means of probabilistic logic always presume the adoption of a rather powerful postulate permitting the selection of a definite truth value in the conclusion and the evaluation of it by means of the probability function. Meanwhile it proves possible to adopt such a postulate in by no means all cases of induction. Even then it must restrict itself to the means of indeterminacy logics.

As N. Rescher has shown [13], certain types of these indeterminacy logics ("quasi-truth-functional systems of propositional logic," to use his terminology) are equivalent to determinacy logics, but such as are multivalued and always have a greater number of truth values than do the corresponding indeterminacy logics. This finding of Rescher's seems to open interesting prospects for investigating the relationship of classes of indeterminacy and multivalued logics. We should also note that since multivalued logics essentially differ from probabilistic the approaches to analysis of induction from positions of indeterminacy and probabilistic logics are substantially different.

One method of constructing an indeterminacy logic may be pictured in the following fashion. A formal system is chosen which is semantically full with respect to some contentful interpretation, but not full in the narrow syntactical sense, that is, it maintains its lack of logical contradiction when its axioms are supplemented by certain formulas (for example, of a type which closely calculates predicates) which are underivable in it. Certain formulas underivable in such a system are further adjoined to it as new axioms, or the addition made is that of new rules of inference which extend the class of formulas derivable in the system.

In this expanded system of logic the formulas which are simultaneously semantically false in a prescribed interpretation may prove to be syntactically derivable. By virtue of this situation the thus constructed formal system will itself be a real instance of an indeterminacy logic. The rules for inference contained in it which, being applicable to true premises, may give both true and false conclusions (but which in no case lead to logical contradiction) -- in Rescher's terminology "non-deductive rules of inference" -- are very reminiscent of the situations of inductive conclusions. Therefore indeterminacy logics of this sort may be employed for analyzing and modeling inductive reasoning.

Thus, for the purpose of analyzing induction Rescher makes use of two indeterminacy logics constructed as expansions of the classic calculation of predicates by introducing a quantor (Mx) in one case ("for most of the individuals in <u>x</u>") and the quantor (Ax) in the other case ("for all the individuals in <u>x</u> about which we have specific information") and by adding the following inference rules, respectively:

Rule 1: $(Mx)Px \vdash (x)Px$.

Rule 1': $(Ax)Px \vdash (x)Px$.

Here Rescher demonstrates that in the case of the first expansion with a plausible supplementary axiom the system as whole proves to be logically contradictory and that Rule 1 must thus be acknowledged to be inadmissible and construction of induction in a system of such an indeterminacy logic to be logically unjustified. The case of the second expansion, however, proves to be correct, Rule 1' is an effectively non-deductive rule of inference, and the indeterminacy logic corresponding to this expansion is a model of some type of inductive reasoning process [14].

Let us note that in a certain expansion of the semantics of such indeterminacy logic, that is, in an expansion which would make all the derivable formulas in our indeterminacy logic true and would turn all nondeductive rules of inference into deductive rules, the whole indeterminacy logic would also be converted into an ordinary deductive logic system. In the application of induction to aproblem this would mean that it was in principle feasible to transform some model of inductive inference, generally a nondeductive one, into a purely deductive model. As instances from the practice of scientific cognition, particularly from mathematics and mathematized natural science, indicate, such a transformation actually takes place and assists in the successful deductive solution of problems which did not yield to pure induction methods. In the general case, however, the algorithm of such a transformation obviously cannot be found for an arbitrary inductive conclusion or, if it can indeed be found, it is by the use of artificial principles of constructing and expanding the semantics and thus by rejection of any natural positions in inductive investigation, that is, of those positions which comply with the real structure of nature. In one way or another, we think, this problem is worth more detailed investigation.

When Rescher's examples of the analysis of inductive inferences by the means of indeterminacy logic are scrutinized it may be noted that this sort of investigation of the structures of different types of induction is found to be more abstract and limited in its possibilities than does an investigation of the structure of induction by probabilistic methods. The specific models of inductive conclusions constructed by such means are as a rule simpler and poorer than the models constructed on a basis of probabilistic logic. And, generally speaking, that is not surprising since the language of probabilistic logic is admittedly richer than the languages of the types of indeterminacy logics examined.

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CYBERNETICS RESEARCH

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STRUCTURE OF DIFFERENTIAL GAMES

[Article by B. N. Pshenichnyy; Moscow, Doklady Akademii Nauk SSSR, Russian, Vol 184, No 2, 1969, pp 285-287]

1. The general structure of a differential game, the payment for which is the completion time of the game, is investigated in the present article. The approach to differential games which is developed below is based on the generalization of the concepts of L. S. Pontryagin discussed as applied to linear games in reference [1].

Let a differential game be described by the system of differential equations

$$\dot{z} = f(z, u, v), \tag{1}$$

where $z \in E^n, u \in E^r, v \in E^s$. The variables u and v vary in the compact sets U and V. In addition, the terminal set M is also given. The game is such that the opponent who has the control u and at his disposal (the opponent U hereafter) tries to remove the phase point z to the set M in the minimum amount of time at the same time as the opponent V tries to prevent this. At each point in time the opponent U knows only the local information about the target, that is, the value of the phase coordinates z at the current point in time. Thus, each of its strategies is a function u(z). Such is the ordinary statement of the problem.

Unfortunately, it leads to a number of great difficulties, and the first of them is that, as a rule, the strategy u(z) must be taken as discontinuous after which the problem of the existence of a solution to system (1) becomes entirely unclear since usually the theorems of the existence of the solution of nonlinear differential equations with discontinuities can be located only with certain restrictions on the nature of the discontinuities. In order to avoid this difficulty, which in some sense is not essentially the matter here, we shall consider that at the initial point in time the opponent V communicates his control in some nonzero time interval σ_1 to the opponent U. By inspection the opponent V selects

the value of $\sigma_1 > 0$ (which is essential). From this information the opponent U constructs his control in the same time interval. As the time σ_1 expires the opponent V reports the time interval σ_2 and his control in it, and so on. Thus, at each point in time the opponent U does not know all the future behavior of the opponent V but only his control in a small time interval. We shall call the described strategies σ-strategies.

We shall state that the game beginning with the point z_0 can be completed at the time T if the opponent U can compare his σ-strategy to each σ-strategy of the opponent V in such a way that the trajectory z(t) of the system (1) corresponding to these controls is in the set M no later than during the time T.

Throughout the entire paper we shall assume that the righthand side (1) is continuously differentiable with respect to z and discontinuous with respect to u and v. In addition, for any z and v the set f(z, U, v) is convex and

$$|zf(z, u, v)| \leqslant c(1 + |z|^2).$$

From the results of reference [2] it follows that under these conditions the set of trajectories of system (1) beginning with the fixed point z_0 is compact in the metric space of the continuous functions for a fixed control v(t) and all possible allowable controls u(t).

2. Definition 1. The operator T_σ, $\sigma \geq 0$ places in correspondence to each set $X \subset E^n$ the set $T_\sigma(X)$ of points $z \in E^n$ such that for each measurable control $v(t), v(t) \in V$, the measurable control $u(t), u(t) \in U$, is found for which the solution of system (1) with the initial condition $z(0) = z$ and $u = u(t)$, $v = v(t)$ falls in the set X no later than after the time σ.

Let us list the properties of the introduced operator which are almost obvious results of its definition.

Property 1. a) $T_\sigma(X) \subset T_{\sigma'}(X)$ when $\sigma' \geqslant \sigma$;

b) $T_\sigma(X) \subset T_{\sigma}(X')$ when $X' \supset X$;

c) $T_0(X) = X$, $T_\sigma(X) \supset X$.

Property 2. $T_{\sigma_1} T_{\sigma_2}(X) \subset T_{\sigma_1+\sigma_2}(X)$.

Property 3. If X is closed, then $T_\sigma(X)$ is closed, and when $\sigma > \sigma_0$, from $z \in T_\sigma(X)$ it follows that $z \in T_{\sigma_0}(X)$.

Property 4. If the closed sets X_i, $i = 1, \ldots,$ are imbedded in each other, that is, $X_{i+1} \subset X_i$, then

$$\bigcap_{i=1}^{\infty} T_\sigma(X_i) = T_\sigma\left(\bigcap_{i=1}^{\infty} X_i\right).$$

Property 5. For arbitrary sets X_α

$$\bigcap_{\alpha} T_\sigma(X_\alpha) \supset T_\sigma(\bigcap_{\alpha} X_\alpha).$$

3. **Definition 2.** The arbitrary finite series of rational numbers τ_i, $\tau_i \leqslant \tau_{i+1}$, $i = 0, 1, \ldots, m$, $\tau_0 = 0$ will be called the rational subdivision

Let us set $|\omega| = \tau_m$. We shall state that the rational subdivision ω' is finer than the subdivision ω (it is denoted by $\omega' \prec \omega$), if $|\omega'| \leqslant |\omega|$ and all the numbers τ_i, $\tau_i \leqslant |\omega'|$, coincide with some of the numbers τ_j' defining the subdivision ω'.

Let for the given ω $\delta_i = \tau_i - \tau_{i-1}$, $i = 1, \ldots, n$, let us define

$$T_\omega(X) = (T_{\delta_m} T_{\delta_{m-1}} \ldots T_{\delta_1}(X).$$

Lemma 1. If $\omega' \prec \omega$, then $T_{\omega'}(X) \subset T_\omega(X)$.

The proof of the lemma follows from property 2 of the operator $T_\sigma(X)$ and definition of the ratio $\omega' \prec \omega$.

Definition 3. $T_t(X) = \bigcap_{|\omega| > t} T_\omega(X)$.

It is obvious that $T_t(X) \subset T_{t'}(X')$, if $t' \geq t$ and $X' \geq X$.

Lemma 2. $T_{t_1+t_2}(X) = T_{t_1} T_{t_2}(X)$.

Proof. We shall denote by ω_σ the rational subdivisions $|\omega_\sigma| > t_1 + t_2$, for which the rational subdivisions ω_1 and ω_2 exist, satisfying the conditions

$$T_{\omega^1} T_{\omega^2}(X) = T_{\omega_\sigma}(X), \quad |\omega^1| > t_1, \ |\omega^2| > t_2. \tag{2}$$

If $|\omega| > t_1 + t_2$, $\omega = \{\tau_0, \tau_1, \ldots, \tau_m\}$, the rational number τ is found such that $\tau > t_2$, $\tau_m - \tau > t_1$. Let us consider the subdivision $\bar{\omega} = \{\tau_0, \tau_1, \ldots, \tau_i, \tau, \tau_{i+1}, \ldots, \tau_m\}$. It is obvious that $\bar{\omega} < \omega$ and

$$T_{\omega^1} T_{\omega^2}(X) = T_{\bar{\omega}}(X) \subset T_{\omega}(X), \tag{3}$$

where $\omega^1 = \{0, \tau_{i+1} - \tau, \ldots, \tau_m - \tau\}$, $\omega^2 = \{\tau_0, \tau_1, \ldots, \tau_i, \tau\}$. Thus, for each subdivision ω a finer subdivision ω_σ will be found. On the other hand, it is possible to place the subdivision ω_σ such that (2) will be fulfilled in correspondance to any two subdivisions ω^1 and ω^2, $|\omega^1| > t_1$, $|\omega^2| > t_2$.

Considering only what has been said, we obtain

$$\tilde{T}_{t_1+t_2}(X) = \bigcap_{|\omega| > t_1+t_2} T_\omega(X) = \bigcap_{|\omega_\sigma| > t_1+t_2} T_{\omega_\sigma}(X) = \bigcap_{|\omega^1| > t_1} \bigcap_{|\omega^2| > t_2} T_{\omega^1} T_{\omega^2}(X) \supset$$

$$\supset \bigcap_{|\omega^1| > t_1} T_{\omega^1}\Big(\bigcap_{|\omega^2| > t_2} T_{\omega^2}(X)\Big) = \tilde{T}_{t_1}\tilde{T}_{t_2}(X), \tag{4}$$

where property 5 of operator T_σ was used in the derivation. Since each subdivision is defined by a set of rational numbers, the number of subdivisions ω, $|\omega| > t_2$, is denumerable. It turns out that it is possible to construct the sequence ω_k, $|\omega_k| > t_2$, $k = 1, \ldots,$ such that $\omega_{k+1} < \omega_k$ and

$$\tilde{T}_{t_2}(X) = \bigcap_{k=1}^{\infty} T_{\omega_k}(X),$$

so that $T_{t_2}(X)$ is formed as the intersection of sets imbedded in each other.

Now

$$\tilde{T}_{t_1}\tilde{T}_{t_2}(X) = \bigcap_{|\omega_1| > t_1} T_{\omega^1}\Big(\bigcap_{|\omega^2| > t_2} T_{\omega^2}(X)\Big) = \bigcap_{|\omega^1| > t_1} T_{\omega^1}\Big(\bigcap_{k=1}^{\infty} T_{\omega_k}(X)\Big) =$$

$$\bigcap_{|\omega^1| > t_1} \bigcap_{k=1}^{\infty} T_{\omega^1} T_{\omega_k}(X) \supset \bigcap_{|\omega_\sigma| > t_1+t_2} T_{\omega_\sigma}(X) = \tilde{T}_{t_1+t_2}(X). \tag{5}$$

Comparison of (4) and (5) completes the proof of the lemma.

4. Theorem. Let the differential game be described by system (1), and let the closed terminal set M be given. Let

$$t(z_0) = \min_{z_0 \in T_t(M)} t.$$

If $z_0 \overline{\in} T_t(M), t \geq 0,$ then $t(z_0) = +\infty.$

Then in order for it to be possible to complete the game from the point z_0 in the time t_0, it is necessary and sufficient that t_0 be less than $t(z_0)$.

Let us indicate the basic ideas of the proof. If $t_0 \geq t(z_0)$, then $z_0 \in T_{t_0}(M)$. If the control v(t) is known in the segment $[0,\sigma]$, than from the relations

$$z_0 \in T_{t_0}(M) = T_\sigma T_{t_0-\sigma}(M) \subset T_\sigma T_{t_0-\sigma}(M)$$

it follows that it is possible to select the control u(t) such that $z(\delta) \in T_{t_0-\delta}(M)$ for some $\delta, 0 < \delta \leq \sigma$. By continuing this process the trajectory of system (1) for any σ-strategy of the opponent V is brought to the set M no later than in the time t_0 since $T_0(M) = M$.

If $t_0 < t(z_0)$, then $z_0 \overline{\in} T_{t_0}(M)$ and the subdivision ω exists such that $z_0 \overline{\in} T_\omega(M), |\omega| > t_0$. Therefore,

$$z_0 \overline{\in} T_{\delta_m} T_{\delta_{m-1}} \ldots T_{\delta_1}(M),$$
$$\delta_m + \delta_{m-1} + \ldots + \delta_1 = |\omega| > t_0.$$

Now using the definition of the operator T_σ, we see that for V the σ-strategy defined by the subdivision and the control of opponent U exists such that however the opponent U operates, the trajectory of the system (1) will not fall in M before the time t_0.

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A PROPERTY OF PERIODIC MOTION OF A SINGLE-CIRCUIT NONLINEAR IMPULSE SYSTEM WITH AN INTEGRATING LINK

[Article by A. S. Alekseyev; Moscow, Doklady Ak'''ii Nauk SSSR , Russian, Vol 184, No 2, 1969, pp 307-308]

Let us investigate the possible periodic motions of a control system the dynamics of which is described by the differential equations

$$\dot{x} = Ax + b\varphi(\mu), \quad \dot{\mu} = \sum_{j=0}^{\infty} (g(t) + c^T x)\,\delta(t - j\tau), \tag{1}$$

where A is a fixed nonsingular matrix (n × n); b and c are fixed column matrices (n × 1); x(t) is the same column of desired functions; $\mu(t)$, $\varphi(\mu)$ and g(t) are the desired and given scalar functions and the given scalar function which is periodic with a period a multiple of τ, respectively. Equations (1) are a special case of equations (5) from [1], and they represent the dynamics of a single-circuit ideal pulse nonlinear control system in the circuitry of which, in addition to common feedback, a summing element, an ideal pulse element operating with a repetition period τ, an ideal integrating element, the time constant of which is selected as the time scale, a static nonlinear element with the nonlinear characteristic $\varphi(\mu)$ and a linear element of order n are connected. From a system with different real pulse elements usually we can convert to a system with ideal elements [2-4].

The solution of system (1) for $x(-0) = x^0$ and $\mu(-0) = \mu^0$ can be written [5,6] in the form

$$x(t) = e^{At}x^0 + \int_0^t e^{A(t-s)} b\varphi(\mu(s))\,ds, \quad \mu(t) = \mu^0 + \sum_{j=0}^{[t/\tau]} (g^j + c^T x^j)\,1(t - j\tau), \tag{2}$$

where we have denoted $g^j = g(j\tau - 0)$ and $x^j = x(j\tau - 0)$, $j = 0, 1, 2, \ldots$

Then denoting $\dot{\mu}^j = \dot{\mu}(j\tau - 0)$ and converting analogously [1] to the point transformation in the phase space corresponding to the motion of the system (1) and realizable with each cycle of the pulse element of the system, we obtain for $j = 1, 2, \ldots$

$$x^j = e^{A\tau}x^{j-1} + (e^{A\tau} - E)A^{-1}b\varphi(\mu^j), \tag{3}$$

$$\mu^j = \mu^{j-1} + g^{j-1} + c^T x^{j-1} = \mu^0 + \sum_{i=0}^{j-1}(g^i + c^T x^i). \tag{4}$$

Using the fact that the matrices A, A^{-1} and $e^{A\tau}$ are pairwise commutative [7], it is not difficult to obtain the indicated point transformation realizable in the time $T = m\tau$ with an arbitrary m from (3) and (4) by the method of induction in the form

$$x^m = e^{mA\tau}x^0 + (e^{A\tau} - E)\,A^{-1}\left(\sum_{j=1}^{m}\varphi(\mu^j)\,e^{(m-j)A\tau}\right)b, \tag{5}$$

$$\mu^m = \mu^0 + \sum_{j=0}^{m-1} g^j + c^T\left(\frac{e^{mA\tau} - E}{e^{A\tau} - E}x^0 - A^{-1}b\sum_{j=1}^{m}\varphi(\mu^j) + \right.$$
$$\left. + A^{-1}\left(\sum_{j=1}^{m}\varphi(\mu^j)\,e^{(m-j)A\tau}\right)b\right), \tag{6}$$

where the variables μ_j are defined by expression (4).

Now if $T = m\tau$ is a multiple of the period of the controlling action $g(t)$ or $g(t) \equiv 0$, then for existence of periodic motion with a period $T = m\tau$ in system (1), the transformations (5) and (6) must have an invariant point which is not the point of least multiplicity; that is, the following relations must be satisfied:

$$x^m = x^0 = \dot{x}, \qquad \mu^m = \mu^0 = \dot{\mu}, \tag{7}$$

which jointly with (5), (6), (3) and (4) determines the coordinates of the image point in this periodic motion at the times $k\tau$, $(k + 1)\tau$, ..., $(k + m - 1)\tau$ for an arbitrary k. The stability of this periodic motion will be determined by the fact that the roots of the characteristic equation belong to a unit circle

$$\det\left|\begin{array}{c:c} \partial x^m/\partial x^0 - zE & \partial x^m/\partial\mu^0 \\ \hdashline \partial\mu^m/\partial x^0 & \partial\mu^m/\partial\mu^0 - z \end{array}\right| = 0, \tag{8}$$

in which the derivatives are taken at an invariant point.

From (5) and (6) it is easy to see that the following property exists.

For periodic motions of a (impulse) nonlinear system (1) with the period $T = m\tau$ under the condition $g(t + m\tau) = g(t)$ the following relation is satisfied:

$$\sum_{j=1}^{m} g(j\tau) = c^T A^{-1} b \sum_{j=1}^{m} \varphi(\mu^j), \tag{9}$$

and under the condition that $g(t) \equiv 0$, that is, for an autonomous system the relation

$$\sum_{j=1}^{m} \varphi(\mu^j) = 0 \tag{10}$$

is satisfied.

This fact emphasizes the symmetrizing role of the integrating link in the system which was pointed out by Ya. Z. Tsypkin in 1953 when discussing the theory of relay systems at the 2nd All-Union Conference on Automatic Control.

In addition, the matrix of the transformations (5) and (6) linearized at the invariant point (7) is the product in reverse order of the matrices of the D_j-single transformations (3) and (4) linearized at the same points. Therefore, the free term of the characteristic equation (8) is equal to det $e^{mA\tau}$. Actually, for $j = 1, 2, \ldots, m$

$$\det D_j = \det \left| \begin{array}{c:c} e^{A\tau} + (e^{A\tau} - E) A^{-1} b \varphi'(\mathring{\mu}^j) c^{T'} & (e^{A\tau} - E) A^{-1} b \varphi'(\mathring{\mu}^j) \\ \hdashline c^T & 1 \end{array} \right| = \det e^{A\tau}, \tag{11}$$

which it is easy to see by substracting the lower row multiplied from the left by the column $(e^{A\tau} - E) A^{-1} b \varphi'(\mu^j)$ from the upper n rows of (11).

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GENERAL CONCAVE-CONVEX GAMES

[Article by N. T. Tynyanskiy; Moscow, Doklady Akademii Nauk SSSR, Russian, Vol 184, No 2, 1969, pp 303-306]

Let the game G = [U; V; f] be given where the set of strategies U and V are arbitrary concave sets lying in the Euclidian spaces E^k and E^l, respectively, and let the nucleus of the game G be a concave-convex function f(u; v), concave and semicontinuous from above with respect to the variable u for each fixed $v \in V$ and convex and semicontinuous from below with respect to the variable v for each fixed $u \in U$ satisfying the closure condition consisting in the fact that any point $(u^0, v^0) \in E^{k+l}$, at which $\overline{\lim}_{u \in U, u \to u^0} f(u; v)$ for all $v \in V$ and $\underline{\lim}_{v \in V, v \to v^0} f(u; v)$ for all finite $u \in U$, belong to the regions of definition of U × V and $f(u^0; v^0) =$

$$= \overline{\lim_{u \in U, u \to u^0}} f(u; v^0) = \underline{\lim_{v \in V, v \to v^0}} f(u^0; v).$$

The situation $(u^0; v^0)$ is called the equilibrium situation (ε-equilibrium) if $f(u^0; v) \geqslant f(u^0; v^0) \geqslant f(u; v^0)$ $(f(u^0; v) + \varepsilon \geqslant f(u^0; v^0) \geqslant f(u; v^0) - \varepsilon)$ for all $(u; v) \in U \times V$.

The method of constructing dual problems of optimization and a dual game proposed in [4] is applied to investigation of the game G. For construction of dual problems it is necessary to represent the function f(u; v) and its region of definition U × V in the form

$$f(u; v) = f_1(u; v) + f_2(u; v), \quad U = U_1 \cap U_2, \quad V = V_1 \cap V_2,$$

where $f_i(u; v)$ is a closed concave-convex function defined on the straight line of the product of the convex sets U_i and V_i ($i = 1, 2$) lying in E^k and E^l, respectively.

This can be done by many procedures. This offers the possibility in each specific case of selecting the most appropriate representation.

A. Let us introduce the following notation. Let

$$f_1(u;v) = f_2(u;v) = {}^1/_2 f(u;v), \quad U_1 = U_2 = U, \quad V_1 = V_2 = V.$$

Then

$$\text{I.} \quad \varphi(\gamma;v) = \varphi_1(\gamma;v) = \varphi_2(\gamma;v) = \sup_{u\in U}[{}^1/_2 f(u;v) + (\gamma, u)]$$

with the region of definition $(\Gamma; V) = (\Gamma_1; V_1) = (\Gamma_2; V_2) = \{(\gamma; v) \in \in E^{k+l} / v \in V, \ \varphi(\gamma;v) < \infty\}$;

$$\psi(u;\delta) = \psi_1(u;\delta) = \psi_2(u;\delta) = \inf_{v\in V}[{}^1/_2 f(u;v) - (\delta, v)]$$

with the region of definition $(U; \Delta) = (U_1; \Delta_1) = (U_2; \Delta_2) = \{(u;\delta) \in \in E^{k+l} / u \in U; \ \psi(u;\delta) > -\infty\}$.

The functions $\varphi(\gamma;v)$ and $\psi(u;\delta)$ are conjugate (see [5]).

II. If the function

$$g'(\gamma;\delta) = g'_1(\gamma;\delta) = g'_2(\gamma;\delta) = \inf_{v\in V}\sup_{u\in U}[{}^1/_2 f(u;v) + (\gamma, u) - (\delta, v)],$$

having finite values in $\Gamma \times \Delta$ is semicontinuous from below with respect to γ or the function

$$g''(\gamma;\delta) = g''_1(\gamma;\delta) = g''_2(\gamma;\delta) = \sup_{u\in U}\inf_{v\in V}[{}^1/_2 f(u;v) + (\gamma, u) - (\delta, v)],$$

also finite in $\Gamma \times \Delta$ is semicontinuous from above with respect to δ, then $g'(\gamma;\delta) = g'(\gamma;\delta) = g(\gamma;\delta)$ is a closed convex-concave function in $\Gamma \times \Delta$.

If in the defining relations for φ, ψ and g the signs in front of the scalar products are changed to the opposite signs, we obtain new functions with their regions of definition $\varphi(-\gamma;v)$, $(-\Gamma;V)$, $\psi(u;-\delta)$, $(U;-\Delta)$, $g(-\gamma;-\delta)$, $(-\Gamma;-\Delta)$.

B. Now let us formulate the following optimization problems.

Problem I. Find $\min_{v\in V}\sup_{u\in V} f(u;v)$.

Problem I'. Find $\max_{u \in U} \inf_{v \in V} f(u; v)$.

Problem II. Find $\min_{(\gamma; v) \in (\Gamma; V) \cap (-\Gamma; V)} [\varphi(\gamma; v) + \varphi(-\gamma; v)]$.

Problem II'. Find $\max_{(u; \delta) \in (U; \Delta) \cap (U; -\Delta)} [\psi(u; \delta) + \psi(u; -\delta)]$.

Problem III. Find $\min_{\gamma \in \Gamma \cap (-\Gamma)} \sup_{\delta \in \Delta \cap (-\Delta)} [g(\gamma; \delta) + g(-\gamma; -\delta)]$.

Problem III'. Find $\max_{\delta \in \Delta \cap (-\Delta)} \inf_{\gamma \in \Gamma \cap (-\Gamma)} [g(\gamma; \delta) + g(-\gamma; -\delta)]$.

The problem in which the optimal value of the purpose function is achieved is called resolvable, and in the case where the optimal value exists but is not achieved, weakly resolvable.

The following statements are true with respect to weak resolvability of the formulated problems.

B_1. If the sets $(\Gamma; V) \cap (-\Gamma; V)$ and $(U; \Delta) \cap (U; -\Delta)$ are not empty, all the formulated problems are weakly resolvable and their optimal values are equal.

B_2. In order that all the problems be weakly resolvable, it is necessary and sufficient that the origin of the coordinates of the space E^{k+l} belong to the set $\Gamma \times \Delta$.

With respect to the resolvability of the problems we have:

B_3. If problems II and II' are resolvable, all the problems are resolvable and their optimal values are equal.

B_4. Problems II and II' are resolvable when and only when problems I and II' are resolvable where all the optimal values are equal.

B_5. If the origin of the coordinates of the states E^{k+l} is an internal point of the set $\Gamma \times \Delta$, all the problems are resolvable and their optimal values are equal.

C. Let us proceed to the problem of the existence of an equilibrium situation in the game G.

Let us set $\tilde{G} = [\tilde{\Gamma}; \tilde{\Delta}; \tilde{g}]$, where $\tilde{\Gamma} = \Gamma \cap (-\Gamma)$, $\tilde{\Delta} = \Delta \cap (-\Delta)$, $\tilde{g}(\gamma; \delta) = g(\gamma; \delta) + g(-\gamma; -\delta)$. Thus, the defined game $\tilde{G}$ is called reciprocal to the game G.

Theorem 1. If the sets $(\Gamma; V) \cap (-\Gamma; V)$ and $(U; \Delta) \cap (U; -\Delta)$ are not empty, the games G and $\tilde{G}$ have situations of ε-equilibrium for any $\varepsilon > 0$.

Theorem 2. The following conditions are equivalent:

1) The game G has the situation of ε-equilibrium for any $\varepsilon > 0$.

2) The game G has the situation of ε-equilibrium for a fixed $\varepsilon > 0$.

3) The expressions $\inf_{v \in V} \sup_{u \in U} f(u; v)$ and $\sup_{u \in U} \inf_{v \in V} f(u; v)$ exist that is, they assume finite values.

4) We have $u^0 \in U$ и $v^0 \in V$ such that the expressions $\inf_{v \in V} f(u_0; v)$ and $\sup_{u \in U} f(u; v_0)$ have finite values.

5) The sets $(\Gamma; V) \cap (-\Gamma; V)$ and $(U; \Delta) \cap (U; -\Delta)$ are not empty.

6) The origin of the coordinates of the space E^{k+l} belongs to the set $\Gamma \times \Delta$.

Theorem 3. The following statements exist:

1) If the problems II and II' are resolvable, the games G and $\tilde{G}$ have a situation of equilibrium.

2) The problems II and II' are resolvable when and only when the game G has the situation of equilibrium.

3) The game G has the situation of equilibrium when and only when the expressions

$$\min_{v \in V} \sup_{u \in U} f(u; v) \quad \text{and} \quad \max_{u \in U} \inf_{v \in V} f(u; v)$$

exist .

The games G and $\tilde{G}$ have the situation of equilibrium, and the problems of mathematical programming reciprocal to them are resolvable in the following cases:

1. The sets $(\Gamma; V) \cap (-\Gamma; V)$ and $(U; \Delta) \cap (U; -\Delta)$ are bounded.

2. The set $(\Gamma; V) \cap (-\Gamma; V)$ $((U; \Delta) \cap (U; -\Delta))$ is bounded and has a relatively internal point.

3. The sets $(\Gamma; V) \cap (-\Gamma; V)$ and $(U; \Delta) \cap (U; -\Delta)$ have internal points.

4. The sets U and V are bounded.

5. The origin of the coordinates of the space E^{k+l} belongs to the set $\Gamma \times \Delta$ and is a relatively internal point of this set.

D. Let us consider the following problem of convex programming.

Let the continuous concave functions $p(u), q_1(u), \ldots, q_l(u)$, defined in a closed convex set $U = \{u \in E^k / u \geqslant 0\}$ be given. Then the problem is stated:

Let us find $u^0 \in U_q = \{u \in E^k / u \geqslant 0, \quad q(u) = (q_1(u), \ldots, q_l(u)) \geqslant 0\}$,

at which the scalar function p(u) reaches its greatest value.

The Lagrange function L(u; v) of this problem has the form $L(u; v) = p(u) + (v, q(u))$ with the closed region of definition U × V where $V = \{v \in E^l / v \geqslant 0\}$.

By construction of L(u; v) there is a closed concave-convex function and all the above-presented results are applicable to it. However, in this case another representation suggests itself, namely:

$$f_1(u; v) = p(u), \; f_2(u; v) = (v, q(u)), \quad U_1 = U_2 = U, \quad V_1 = V_2 = V.$$

Carrying out the constructions analogous to the constructions of items A and B (see also [4]), we obtain two dual convex problems of optimization and a dual game. Let us present the corresponding relations for φ_i and ψ_i $(i = 1, 2)$:

$$\varphi_1(\gamma; v) = \sup_{u \in U_1} [f_1(u; v) + (\gamma, u)] = \sup_{u \in U} [p(u) + (\gamma, u)],$$
$$(\Gamma_1; V) = \{(\gamma; v) \in E^{k+l} / v \geqslant 0, \quad \varphi_1(\gamma; v) < \infty\},$$
$$\varphi_2(-\gamma; v) = \sup_{u \in U_2} [f_2(u; v) - (\gamma, u)] = \sup_{u \in U} [(v, q(u)) - (\gamma, u)],$$
$$(-\Gamma_2; V) = \{(\gamma; v) \in E^{k+l} / v \geqslant 0, \quad \varphi_2(-\gamma; v) < \infty\},$$
$$\psi_1(u; \delta) = p(u), \quad (U; \Delta_1) = \{(u; \delta) \in E^{k+l} / \delta \geqslant 0, \quad u \geqslant 0\},$$
$$\psi_2(u; \delta) = 0,$$
$$(U; -\Delta_2) = \{(u; \delta) \in E^{k+l} / q(u) + \delta \geqslant 0, u \geqslant 0\}.$$

Problem II'a. Find

$$\max_{(u;\,\delta)\in(U;\,\Delta_1)\cap(U;\,-\Delta_2)} [\psi_1(u;\,\delta) + \psi_2(u;\,-\delta)].$$

This problem is equivalent to the initial problem.

Problem IIa. Find

$$\min_{(\gamma;\,v)\in(\Gamma_1;\,V)\cap(-\Gamma_2;\,V)} [\varphi_1(\gamma;\,v)+\varphi_2(-\gamma;\,v)].$$

Problem IIa will be called reciprocal to the initial problem on the basis of the Lagrange function. The duality of problems II'a and IIa is the duality occurring when using conjugate functions (see, for example, [1], Chapter 7).

Theorem 4. The Lagrange function L(u; v) has a saddle point (the situation of equilibrium) when and only when the initial problem of convex programming and the problem reciprocal to it on the basis of the Lagrange function are resolvable and have equal optimal values (compare [3, 6]).

The full proof of the presented results appears in [7].

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RELIABILITY OF LOGICAL CIRCUITS WITH FEEDBACK

[Article by A. Kh. Giorgadze; Tbilisi, Soobshcheniya Akademii Nauk Gruzinskoy SSR, Russian, Vol 52, No 2, 1968, pp 315-318]

Let us consider the logical circuit L having m binary inputs, one output and a feedback circuit connecting the output L to one of its elements [1].

It is convenient to represent the functioning of L with the help of the transition chart of the Muhr automaton in the following way.

Let us introduce two states corresponding to values of 1 and 0 in the output channel of L. Let α_i be a letter of the input alphabet, $i = 1, 2, \ldots, 2^m$; $P(\alpha_i)$ be the probability of occurrence of the letter α_i. The probabilities $P(\alpha_i)$ are independent of time even in the aggregate.

The transition matrix of an ideally operating circuit L when the letter α_i is input to it will be denoted by A_i^0, and we shall compile the generalized matrix $A^0 = \sum_i A_i^0 \cdot P(\alpha_i)$. The transition matrix of the circuit L permitting failures will be denoted by A_i and we shall introduce $A = \sum_i A_i P(\alpha_i)$. Finally, let B^j be the matrix of the circuit L^j obtained when the element α_j fails, $j = 1, 2, \ldots, N$; N is the number of elements of L.

The representation of L in the form of an automaton is illustrated by an example (Figure 1,a, 1,b).

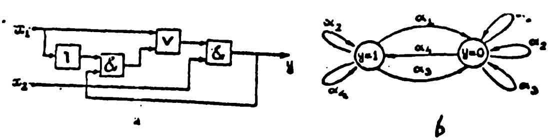

Figure 1

In Figure 1,a we have a logical circuit with a feedback circuit and in Figure 1,b, its automaton representation. For values of the input variables $x_1 = 0$ and $x_2 = 0$ (that is, the letter α_1) and with $y = 1$ at the output (the automaton is in the state $y = 1$) the output value in the mixed cycle will be 0 (the automaton is converted into state $y = 0$). The letters α_2, α_3 and α_4 are tested analogously.

We shall consider the average number of cycles of proper operation T as the desired reliability chararacteristic of the circuit L under the assumption that from the beginning of operation to the time of occurrence of an incorrect result at the system output one failure can occur. No limit is imposed on the multiplicity of failures.

Let us write the condition of logical equivalence of operations of the ideal automaton L and the automaton L' subject to failures:

$$y\bar{y}' \vee \bar{y}y'.$$

Here y and y' are the output functions of the ideal automaton and the automaton with failures respectively.

Let us compile the matrix B_1 describing the Markov chain, the set of states $(s_1, s_2, \ldots, s_k)$ of which is the cartesian product of the sets of states of the automats L and L'. For example (1,a, 1,b) this set is (0, 0), (0, 1), (1, 0), (1,1). The element $P(s_is_j)$ of the matrix B_1 is the probability of simultaneous conversion of the automaton L from state i to state j and the automaton L' from state l to state k. State i of automaton L and l of automaton L', the pair $(i, l) = S_i$, form the state s_i. The pair $(j, k) = s_j$ forms s_j; $i, j, k, l = 1, 2, \ldots, n$; $s_is_j = 1, 2, \ldots, n^2$.

Let us convert the matrix B_1 into B_1^0 in the following way: let us introduce the additional state s_0 into the set $(s_1, s_2, \ldots, s_k)$. In addition, we shall isolate the set of states $C = \{s_{i_1}, s_{i_2}, \ldots, s_{i_d}\}$ such that all $s_j \in C$ are formed by pairs of the type $s_j = (i, i)$. The probabilities $P(s_is_j)$ for $s_j \in C$ and $s_i = 1, 2, \ldots, k$ are the probabilities of transition of the automatons L and L' to identical states.

For the investigated example P(0, 0) and P(1,1). In the matrix B_1^0 all the probabilities $P(s_is_j), s_j \in C$; all the probabilities $P(s_js_l), s_j \in C, = 1, 2, \ldots, k$; $P(s_0s_l), s_l = 1, 2, \ldots, k$ will be set equal to zero. The elements $P(s_is_0)$, $s_i = 1, 2, \ldots, k$ will be set equal to $1 - \sum_{s_j \in C} P(s_is_j)$; the element $P(s_0s_0) = 1$.

Thus, the state s_0 is the absorbing state for the network described by B_1^0. It is easy to see that $\sum_{s_j \in C} P(s_i s_j)$ is the probability of transition of the automata L and L' to identical states from the states froming s_1. The analogous sum of the t-th degree elements of the matrix B_1^0 is the probability that in t cycles the automata L and L' will make the transition from the states forming s_1 to one and the same sequence of states. Thus, the matrix B_1^0 characterizes the reliability of the circuit L permitting failures.

It is possible to obtain the matrix B_2^0 analogously for the reliability characteristic of an automaton with a failure in one of the elements. Let a failure occur in the element a_j in the t-th operating cycle of the automaton L. Then the automaton L degenerates into the automaton L^j which differs from l in that the function $\Psi(a_j)$ of the element x_j in the formulas describing the operation of the automaton with a failure is replaced by $\Psi^*(a_j) \neq \Psi(a_j)$. Performing these operations on the matrices of the automata L and L^j, we obtain the matrix B_2^0.

Let us assume that it is given that $\Phi_j(t)$ is the probability of occurrence of a failure in the element a_j in the t-th cycle. Let us denote the probability that the automata L and L' will pass through the same sequence of states during the course of t cycles by f_1^t, and by f_2^t let us denote the probability that the automata L and L^j will pass through the same sequence of states. In addition, let the probability that the automata L and L^j will be in the states i and j during the cycle t be $Q^t[s_j(i, j)]$ under the condition that the initial state of the automaton L is given.

Then the probability of proper operation of an automaton subject to breakdowns and failure in a_j during the course of r cycles be written in the form

$$R(r) = \sum_{t=1}^{r-1} \sum_{s_j=1}^{k} f_1^t \cdot f_2^{r-t} \cdot \Phi_j(t) \cdot Q^t(s_j)$$

and the average number of cycles of proper operation by

$$T = \sum_{r=1}^{\infty} r R(r).$$

It is possible to calculate the matrices B_1^0 and B_2^0 and also $Q^t(s_j)$ with the help of the stochastic matrices A^0, A, B^j of the automata L, L', L^j using, for example, the method of calculating the reliability of logical nets presented in [2]. In the case where there are d feedback circuits in the logical circuit, the functioning of the logical circuit is represented in the form of an automaton with 2^d states.

It is convenient to use the described procedure for finding T for automata with a "small memory," in particular, for logical circuits with a small number of feedback circuits.

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DISTRIBUTION OF RESOURCES IN AGRICULTURAL PRODUCTION BY THE TABULAR METHOD

[Article by O. K. Aburdzhaniya; Tbilisi, Soobshcheniya Akademii Nauk Gruzinskoy SSR, Russian, Vol 52, No 2, 1968, pp 319-324]

In agricultural production the possibility of performing each operation is limited to a defined calendar time interval $[\tau_i, \tau_j]$ which will be called the tentative time interval [1].

Let us assume that the time required to perform the given operation is equal to the tentative time interval of this operation. Then the occurrence of its initial and final times coincides with the calendar dates τ_i and τ_j, respectively. These moments will be called events, and they will be denoted provisionally by the natural numbers P_i and P_j such that the number of the event corresponding to the end of the given operation will be equal to the number of the initial event directly following it with respect to the operating process. For example, if the operation (P_1, P_2) is given and the operation (P_3, P_4) follows it directly in the process, then it must happen that $P_2 = P_3$. However, it does not always follow from this equality that $\tau_2 = \tau_3$ since operations which follow each other in a defined time interval are often performed in parallel, that is, $\tau_3 < \tau_2$. In other cases $\tau_2 > \tau_3$, that is, there is a defined time interval between the end of the preceding operation and the beginning of the operation following directly after it in the process.

If the operations $\{(P_i, P_j)\}$ are given in tabular form, the numbers $P_j (j = 1, 2, \ldots, n)$ permit us to find their process sequences in this table.

Let Table 1 be given. In columns P_i and P_j of this table we have the numbers of the initial and final events, respectively, corresponding

to the operations $\{(P_i, P_j)\}$. In columns τ_i and τ_j we have the initial and final calendar dates of the tentative time intervals of these operations, respectively. In columns a_{ij} and b_{ij} we have the optimistic and pessimistic estimates [1] which the responsible agents give. In column V_{ij} we have the volumes of executed operations and in column α_{ij}, the daily output norms per unit of required resources. The problem is to arrange the distribution of resources on the basis of these tabular data so that all the given operations will be performed in the tentative time interval.

Table 1

P_i	P_j	τ_i	τ_j	a_{ij}	b_{ij}	V_{ij}	α_{ij}
P_1	P_2	τ_1	τ_2	a_{12}	b_{12}	V_{12}	α_{12}
						$\vdots$	
P_i	P_j	τ_i	τ_j	a_{ij}	b_{ij}	V_{ij}	α_{ij}
							.
P_m	P_s	τ_m	τ_s	a_{ms}	b_{ms}	V_{ms}	α_{ms}

Algorithms

Stage 1. For the solution of the basic problem it is required that the events be numbered so that for any operation (P_i, P_j) we have $i > j$ if P_i is the number of the initial event of the operation and P_j is the number of the final event of this operation. This numbering will be carried out by the Ford algorithm [2]. New event numbers will be written, respectively, in columns P_i and P_j of Table 2.

Table 2

P_i	P_j	t_{ij}	τ'_j	τ''_i	τ''_j	$\varkappa_{ij}$	z_{ij}	$\tau^{\bullet}_i$	$\tau^{\bullet}_j$	$\varkappa^{\bullet}_{ij}$	ψ
1	k	t_{1k}	τ'_k	τ''_1	τ''_k	$\varkappa_{1k}$	z_{1k}	$\tau^{\bullet}_1$	$\tau^{\bullet}_k$	$\varkappa^{\bullet}_{1k}$	N_1 N_k
ℓ	s	$t_{\ell s}$	τ'_s	τ''_ℓ	τ''_s	$\varkappa_{\ell s}$	$z_{\ell s}$	$\tau^{\bullet}_\ell$	$\tau^{\bullet}_s$	$\varkappa^{\bullet}_{\ell s}$	$N_m \cdot N_n$
m	n	t_{mn}	τ'_n	τ''_m	τ''_n	$\varkappa_{mn}$	z_{mn}	$\tau^{\bullet}_m$	$\tau^{\bullet}_n$	$\varkappa^{\bullet}_{mn}$	N_ℓ N_k

Stage 2. The time for execution of the operation is the probability variable which is calculated by the following empirical formula [3]:

$$t_{ij} = \frac{3b_{ij} + 2a_{ij}}{5}. \quad (1)$$

k-th Step. Using the k-th rows of columns a_{ij} and b_{ij} (Table 1), by formula (1) we can calculate the value of t_{ij} and write it in the k-th row of the column t_{ij} (Table 2).

Stage 3. After introducing the variables t_{ij} our assumption loses meaning, since $t_{ij} \leq t_j - t_i$. If we assume that the operation (P_i, P_j) begins with the date τ_{ij}, it ends with the date $\tau'_j = \tau_j + t_{ij}$.

k-th Step. Let us add the element of the k-th row of the column t_{ij} (Table 2) to the element τ_k, and we obtain the element of the k-th row of the column τ'_j.

Stage 4. If the operation (P_k, P_e) follows the operation (P_n, P_m) directly in the process, then the dates of occurrence of the initial events are equal to τ_n and τ_k respectively and the dates of occurrence of the final events are τ'_m and τ'_e respectively, then the following relations must occur:

$$\tau_n \leqq \tau_k \quad (a), \qquad \tau'_m \leqq \tau'_e. \quad (6)$$

After introducing the variables t_{ij} the relation (b) can be violated and it is found that the operation (P_k, P_e) it is found that the operation (P_k, P_e) ends earlier than the directly preceding operation (P_n, P_m), which is impossible in practice. Therefore, the time parameters of the operation (P_k, P_e) must be corrected:

$$\tau''_k = \tau_k + (\tau'_m - \tau'_e); \quad \tau''_e = \tau'_m.$$

k-th Step. If the element S of the k-th row of column P_j (Table 2) corresponds to the operation (a, S), we compare it with all the elements of column P and select the elements $S_1, S_2, \ldots, S_k$ equal to it. They correspond to the operations $(S_1 a_1), (S_2 a_2), \ldots, (S_k a_k)$. Then the element τ_s' of the column τ_j' will be compared with the elements of the same column -- $\tau_{a1}', \tau_{a2}', \ldots, \tau_{ak}'$. If it turns out that

$$\tau_s \leq \begin{cases} \tau'_{a1}, & \text{for } S = S_1, \\ \tau'_{a2}, & \text{for } S = S_2, \\ \tau'_{ak}, & \text{for } S = S_k, \end{cases}$$

the relation (b) is not violated and the elements of the columns τ_i'' and τ_j'' (Table 1) corresponding to the operations (a, S), (S_1, a_1), (S_2, a_2), ..., (S_k, a_k), will be carried over unchanged to the columns τ_i'' and τ_j''. If at least one of the elements $\tau_{as}' < \tau_s'$, we add an element of the column τ_i corresponding to the operation (S_k, a_k) to the difference $(\tau_s' - \tau_{ak}')$, and we write it in the column τ_i'', and for the same operation we write τ_s'' in the column τ_j'.

Stage 5. Since $\tau_j^{\circ} \leq \tau_j$, sometimes there is a time reserve left for the operation (P_i, P_j) which we shall call the absolute time reserve:

$$\mu_{ij} = \tau_j - \tau_j^{\circ}.$$

k-th Step. Let us substract the element of the column τ_j'' of the k-th row from the element of the column τ_j (Table 1) and the k-th row. The value obtained will be written in the k-th row of the column μ_{ij}.

Stage 6. If the executed volume V_{ij} and the calculated probable time of its execution t_{ij} are given for the operation (P_i, P_j), then the intensity of the operation (the daily resource norm) for any n-th type of resources will be

$$r_{ij}^{(n)} = \frac{V_{ij}}{\alpha_{ij}^{(n)} \cdot t_{ij}},$$

where $\alpha_{ij}^{(n)}$ is the part of the volume of operations which a unit of resources of the n-th type will execute per unit time. This quantity is known.

k-th Step. The elements of the k-th row of columns t_{ij} and α_{ij} will be multiplied by each other, and we shall divide the element of the k-th row of the column V_{ij} by the product obtained. The result obtained will be written in the k-th row of column r_{ij}.

Stage 7. Let $A^n(t)$ be the daily norm of the n-th type of resources in the given agricultural production. Let us sum the required quantity of n-th type resources on an arbitrary elementary φ-front (1), and let us denote this sum by $R_\varphi^{(n)}$.

The front on which the relation

$$R_\varphi^{(n)} \leq A^n(\tau),$$

is fulfilled will be called the normal front, and in the opposite case, the critical front. Our problem is to convert the critical front into the normal front if this is possible.

I Step. Let us find the first elementary front: let us find the elements of the columns τ_i" and τ_j' $\tau_0 = \min\{\tau_i'\}$ and $\tau_1 = \min\{\tau_{ij}', \tau_j'\} > \tau_0$, which gives the first elementary front $\{\tau_0, \tau_1\}$. The operations performed on this front will be operations the dates of occurrence of the initial events of which are τ_0. Let us sum the intensities of these operations, and if the relation (*) occurs, then, the elementary front will be normal and the elements of the columns τ_i", τ_j" and μ_{ij} will be carried over without change to the columns τ_i^*, τ_j^* and μ_{ij}^*. If the relation (*) is violated, that is, the elementary front is critical, then we find the distribution of resources for which the relation (*) will be satisfied. Let us number the operations of this front as follows: the first numbers will be assigned to the operations which have operations following directly in the process on this front (this can be defined as follows: the elements of the column P_j of the selected operation will be compared with the element of column P_i; if any element of column P_i' is equal to any element of column P_j, the operation corresponding to the latter element will be assigned the first number). The next numbers will be assigned to the operations in increasing order:

$$[\mu_{ij} - (\tau_1 - \tau_0)] > 0 .$$

From the first elementary front, the operations which have large numbers will be carried over to the second front located directly to the right so that the relation (*) will be satisfied. It is clear that the initial and final dates of the transferred operation will be

$$\tau_i' + (\tau_1 - \tau_0) \quad \tau_j' + (\tau_1 - \tau_0)$$

respectively.

φ-th Step. Let us find the φ-th elementary front: let us find the elements equal to $\tau_{\varphi-1} = \min\{\tau_i'', \tau_j'\}$ and $\tau_\varphi = \min\{\tau_i'', \tau_j'\} > \tau_{\varphi-1}$, which give the φ-th elementary front $[\tau_{\varphi-1}, \tau_\varphi]$. The following relations will occur for the executed operations of this front:

$$\tau_j'' \leqq \tau_{\varphi-1}, \quad \tau_{\varphi-1} \leqq \tau_j' \geqq \tau_\varphi.$$

If the given front is critical, the operations will be transferred from it by the above described rule. The difference lies only in numbering the operations: the first numbers are assigned to the operations beginning in the preceding front, and the numbering of the remaining operations is analogous to the first front.

This is the same for all remaining fronts.

The operations in agricultural production are performed on one or several fronts. The numbers of these fronts will be introduced into the φ-th column of Table 2, and this will completely fill out the given table. The above-stated problem is completely solved, the resources are distributed, and the calendar dates of the beginning and end of the operations are calculated.

However, we must consider that after operation of the algorithm critical fronts can still remain. In this case it is necessary to reexamine the plan: either the plan cannot be fulfilled under the given conditions and it must be corrected or it is necessary to introduce additional quantities of resources. These problems are solved directly by management (the board of directors and administration) depending on the existing situation.

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